

# Financial Crises, Recoveries and Labor Market Dynamics: Evidence from a Data-Rich DSGE Model

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## Abstract

I investigate the extent to which modern Dynamic Stochastic General Equilibrium (DSGE) models can produce macroeconomic and labor market dynamics in response to a financial crisis that are consistent with the experience of the Great Recession. Using the methods of Boivin and Giannoni (2006) and Kryshko (2011), I estimate two DSGE models in a data-rich environment. This allows me to examine the dynamics of economic series not obtainable in traditional DSGE model estimation. I find that negative financial shocks are associated with longer recoveries in real investment, capital intensive sectors of the labor market and average unemployment duration when compared to other negative output shocks. These results hold when the decline in output is normalized across the shocks. The two models estimated in this paper include close variations of the Smets & Wouters (2003, 2007) New Keynesian model and the FRBNY (Del Negro et al. 2013) model that augments the Smets & Wouters model with a financial accelerator. I find the model with a financial accelerator that is estimated in a data-rich environment is equipped with better tools to identify the dynamics associated with the Great Recession and its recovery in regard to core macroeconomic variables and many labor and financial metrics including the unemployment rate, total number of employees by sector and business loans.

**Keywords:** Data-rich DSGE, Financial accelerator, DSGE-DFM, Bayesian estimation

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# 1 Introduction

Modern day macroeconomic theory has greatly leaned on structural Dynamic Stochastic General Equilibrium (DSGE) modeling. These models give policymakers a workshop in which co-movements of aggregate macroeconomic time series can be evaluated over the business cycle. The Smets and Wouters (2003, 2007) model (SW) in particular is widely considered the “workhorse” of the DSGE literature. However, Del Negro and Schorfheide (2012) have found this model to be limited in identifying the financial crisis for most of 2008, including the 4th quarter of 2008 when the crisis was in full swing. A model that was able to identify the Great Recession six months earlier than the SW model is a variant of the SW model with financial frictions (SWFF). The SWFF model introduces a Bernanke, Gertler and Gilchrist (1999) financial accelerator mechanism and closely follows the entrepreneurial sector of the DSGE model of Christiano et al. (2010) and the FRBNY model outlined by Del Negro et al. (2013). Del Negro and Schorfheide (2012) compared the SW and SWFF models forecasting performance over the past two decades when the models were estimated under a standard set of seven or eight data series. They found that during the Great Recession the modified SWFF model was better at forecasting output and inflation when compared to the original SW model.

Given the construction of traditional DSGE model estimation (DSGE-Reg) economists are limited to comparing the two models on only a handful of co-movements among these aggregate series. However, the techniques of Boivin and Giannoni (2006) and Kryshko (2011) provide an avenue through which DSGE environments can be used to study such series as the unemployment rate, unemployment duration and employees by sector even when no such series are directly incorporated into the structural model. The Boivin and Giannoni (2006) technique (DSGE-DFM) allows DSGE models to be estimated using a large data vector of macroeconomic time series. The series that are not directly incorporated inside the DSGE model are allowed to load on economic variables and structural processes that are inside the DSGE model. The estimated structural parameters and loadings allow me to examine the dynamic effects of the structural shocks inside the DSGE model as well as the dynamics of additional data series important to the questions of this paper and the policymaker.

This method has been most recently used by Gali et al. (2012), Brave et al. (2012) Justiniano et al. (2013) and Barsky et al. (2014) who have all expanded the observable vector to improve the identification of unobservable and observable states and thus improve the estimation of the structural parameters. Gali et al. (2012) and Justiniano et al. (2013) promotes the use of multiple series for the measurement of wages, while Brave et al. (2012) and Barsky et al. (2014) uses multiple measures of inflation to estimate their perspective models. However, these papers used the method to allow for multiple data variables measuring the same model concepts and I will use the methodology to allow a large vector of macro-financial data to load on all DSGE model states.

In this paper, I estimate both the SW and SWFF models using the DSGE-DFM method. The macro-financial time series I use to conduct these estimations is a near replica of the Stock and Watson (2003) dataset used in estimating their Dynamic Factor Model. It includes labor and financial data series that are usually not utilized in DSGE-Reg estimation. These include unemployment rates and durations employment by sector, stock price indexes, housing starts and many price and wage indexes beyond the standard CPI index and GDP deflator.

This approach allows me to empirically examine the question of why some recessions are associated with jobless or wageless recoveries and others are not. In particular, I investigate whether recently developed (and popular) structural models of the U.S. economy can generate labor market dynamics similar to those seen in the data. To explore the economic and labor market effects of various exogenous shocks I examine structural impulse response functions (IRF's) for series that are usually not inside DSGE models. Many of these IRF's are only obtainable if embedded in a dynamic factor model with little or no theoretical interpretation of the original shock by which they are generated. However, the DSGE-DFM estimation technique creates a structural foundation of what type of initial shock has created the disturbance.

After estimating both models in a data-rich environment, I calibrate the SWFF model to ensure that all shocks decrease real GDP by the same amount. I find evidence that financial shocks (corresponding to an increased spread between the risk and risk-free interest rates inside the model) are associated with higher levels of unemployment and longer average

unemployment duration in comparison to responses to other types of shocks with identical output decreases. These results suggest that the relationship between unemployment and GDP growth implied by Okun's Law might be state-dependent. I also find that sectors associated with more capital intensive operations (manufacturing and construction sectors) are the very sectors that are slowest to recover from a financial shock. Labor market series are not the only series where such a pattern exists, decreases in real investment, residential investment, exports and new orders are larger and last longer in response to negative financial shocks when compared to negative consumer, monetary, or supply shocks.

Finally, I closely examine the period surrounding the Great Recession and its recovery. I conduct simulations and forecasts for 2008Q3, 2008Q4 and 2009Q1 of both DSGE-DFM models. I find that the SWFF-DFM model was able to foresee the decrease in the number of overall jobs, number of jobs in the manufacturing and construction sectors and the rise in the unemployment rate. In comparison to the SW-DFM model, the SWFF-DFM model was able to predict these declines earlier and more accurately. These results suggest that the SWFF model estimated in a data-rich environment would have predicted the labor market dynamics associated with the Great Recession and its proceeding recovery. I also find that many of the in-sample forecasts of such variables do not differ from each other in tranquil economic times. It is only in times of financial volatility that I see the simulated paths from the two models begin to differ. These results extend to core macroeconomic variables as well, I find that the dynamics associated with GDP and consumption growth during the recovery can also be predicted best by the SWFF DSGE-DFM model.

The results of the paper are consistent with other empirical work that suggests that sluggish labor market recoveries may be directly linked to what initiated the preceding recession. In particular, Boeri et al. (2012) used firm-level balance sheets and employment records and found that firms in industries that use more temporary financing in everyday business operations adjust employment levels much more when credit shocks decrease liquidity than firms with less financing on their balance sheet. This liquidity channel leads to larger job losses and slower hiring when a decrease in economic output is caused by a financial shock rather than a demand or supply shock. A result that is also found by Chodorow-Reich (2013) and Duygan-Bump et al. (2015) using firm-level data.



The relationship of job destructions and liquidity is not only found at the firm-level but is also seen at the aggregate labor market level. Calvo et al. (2012) studied economic data from thirty-five emerging and advanced economies and found that the unemployment rate rose higher and remained higher for longer periods of time in recessions caused by financial shocks when compared to recessions caused by productivity shocks. Calvo et al. (2012) also examined wage dynamics and found that financial recessions can be associated with either jobless recoveries or “wageless” recoveries depending on the level of inflation observed in the economy during the recovery period. Schmitt-Grohe et al. (2017) finds that negative demand and financial shocks that push the economy to the zero lower bound further slow down recoveries and that employment growth can remain low even as productivity and output growth returns to their long-run levels.

In addition to these papers, my paper also fits into the structural DSGE literature of labor market dynamics around the Great Recession. Gali et al. (2012), Christiano et al. (2015), Christiano et al. (2016) incorporate a more advanced labor market in their perspective DSGE models than either the SW or SWFF model. The models of Gali et al. (2012) and Christiano et al. (2015) are able to simulate and/or forecast the dynamics of employment, unemployment and other aggregate labor market statistics quite nicely as does the SWFF-DFM model of this paper. However, the SWFF-DFM model is able to also capture the labor market and output dynamics of less aggregate statistics, such as employment and production by sector.

The remainder of this paper is structured as follows. Section 2 explains each agent of the economy and the linearized equations for both models needed to replicate the results of this paper. Section 3 outlines the estimation technique used to incorporate the large set of economic and financial series including the adaptive Metropolis-within-Gibbs algorithm used in estimating both models in the data-rich environment. Also included in this section is a description of the priors for the state-space and structural parameters as well as an overview of the data series and how they were collected, transformed, and grouped. Section 4 discusses the dynamics of the SWFF-DFM model including estimated IRF’s for different “types” of normalized output declines induced by the various structural shocks inside the SWFF model. Section 5 shows the simulated paths of both the SW-DFM and SWFF-DFM

models for various labor, output and finance series around the trough and recovery of the Great Recession. Section 6 concludes and discusses future extensions.

## 2 The DSGE Models

I consider two DSGE models in this paper, the first model is based on the FRBNY model outlined by Del Negro et al. (2013). This model is an extension of the Smets and Wouters (2003, 2007) New Keynesian model with the addition of a credit market with frictions that closely follows the financial accelerator model created by Bernanke, Gertler and Gilchrist (1999). It incorporates many of the features of Christiano, Motto and Rostagno (2010). The second model has no credit channel and closely follows the Smets and Wouters (2003) model. This model will be referred to as SW while the model with financial frictions will be referred to as SWFF. In this section, I first outline the agents in the SWFF model and I present the linearized equations of the model around the steady state that I use to produce my results. Finally, I introduce the components of the SW model that differ from the SWFF model, as well as any linearized equations that change as a result of how the SW model is microfounded.

### 2.1 General Outline of SWFF Model

The model involves a number of exogenous shocks, economic agents, and market frictions. The agents include households, intermediate and wholesale firms, banks, entrepreneurs, capital producers, employment agencies, and government agencies.

**Households** supply household-specific labor to employment agencies. Households maximize a CRRA utility function over an infinite horizon with additively separable utility in consumption, leisure and money. Utility from consumption has habit persistence as it is realized by a relative measure of total consumption in the last time period. Labor is differentiated over households, and is not perfectly competitive implying households hold some monopoly power over wages. The model includes sticky nominal wages set in a Calvo (1983) manner with wage indexation to those who can not freely optimize their wage. In addition to holding money, households can save in Government bonds and/or deposits in banks.

Households are subject to an exogenous preference shock that can be viewed as a shock in the consumer's consumption and saving decisions.

**Employment Agencies** package and sell labor bought from the household to intermediate-firms. Employment agencies are perfectly competitive but must buy specialized labor from households who hold some monopoly power over wages. Households and Employment Agencies may only renegotiate wages with a certain probability but are subject to inflation indexation. Employment agencies are subject to wage mark-up shocks that capture exogenous changes in the monopolistic power households hold over their specialized labor.

**Firms** come in two forms, intermediate good producing firms and final good producing firms. There is a continuum of intermediate good firms, who supply intermediate goods in a monopolistically competitive market. Intermediate firms produce differentiated goods, decide on labor and capital inputs, and set prices in a Calvo-like manner. As with wages, those firms unable to change their prices, are able to partially index them to past inflation rates. Intermediate firms face two exogenous shocks, the first is a productivity shock that affects their production ability and the second is a price mark-up shock. The price mark-up shock captures the degree of competitiveness in the intermediate goods market. Final goods use intermediate goods in production and are produced in perfect competition. The final good is sold to the households and capital producers in the form of consumption.

**Capital Producers** buy consumption output from the final goods sector and transform it into new capital. The creation of new capital (Investment) requires both the newly bought consumption output and the previous stock of capital in the economy which they buy from entrepreneurs. The investment procedure is subject to convex adjustment costs making it more expensive to produce more capital in times of large investment growth. Capital producers are subject to investment shocks that affect the marginal efficiency of investment as in Justiniano et al. (2011).

**Financial Sector** centers around two economic agents, banks and entrepreneurs. Entrepreneurs enter the period with some level of net worth. They must use their net worth and an agreed upon loan from the bank to buy capital from the capital producers. Once the capital is bought they are affected by an idiosyncratic risk shock that can decrease or increase their overall level of capital just purchased. The entrepreneur must then decide the

utilization of the new level of capital and rent it out to intermediate firms to be used in their production process. Once the capital has been used in the production process the non-depreciated capital is purchased by the capital producers. If entrepreneurs received enough revenue they pay back the agreed upon loan with interest to the bank. If entrepreneurs do not have enough revenue a proportion of their revenue is seized by the bank. Banks incorporate the risk of default by charging entrepreneurs an interest rate higher than the deposit rate payed to households.

**Government Agencies** are comprised of a monetary authority and a fiscal authority. The short term nominal interest rate is determined by the monetary authority, which is assumed to follow a generalized Taylor Rule and is subject to monetary policy shocks. The monetary authority supplies the corresponding money demanded by the household to support the targeted nominal interest rate. The fiscal authority sets government spending and collects lump sum taxes. It is subject to exogenous government spending shocks.

## 2.2 Log Linear Equations

The model is linearized around the non-stochastic steady state and then solved using the Sims (2002) method. This solution is the transition equation in the state-space set-up of Section 3. Variables denoted with a hat are defined as log deviations around the steady state.  $\left(\hat{Y}_t = \log\left(\frac{Y_t}{Y}\right)\right)$  Variables denoted without a time script are steady state values. In all, the model is reduced to 12 equations and eight exogenous shocks all of which are listed in this subsection.

Physical capital  $\bar{K}_t$  accumulates according to:

$$\hat{\bar{K}}_t = (1 - \tau)\hat{\bar{K}}_{t-1} + \tau\hat{I}_t + \tau(1 + \beta)S''\hat{\varepsilon}_t^I \quad (2.1)$$

where  $\varepsilon_t^I$  is an AR(1) investment shock and  $\tau$  is the depreciation rate and  $S''$  is a parameter that governs investment adjustment costs. A large  $S''$  implies that adjusting an investment schedule is costly.

Labor Demand is given by

$$\hat{L}_t = -\hat{w}_t + (1 + \frac{1}{\psi})\hat{r}_t^k + \hat{K}_{t-1} \quad (2.2)$$

where  $r_t^k$  is the real rental rate of capital and  $\psi$  is a parameter that captures utilization costs of capital. A large  $\psi$  infers that capital utilization costs are high. The economy's resource constraint and production function take the form:

$$\hat{Y}_t = C_y \hat{C}_t + I_y \hat{I}_t + \frac{r^k \bar{k}_y}{\psi} \hat{r}_t^k + \mathcal{M}_t + \varepsilon_t^G \quad (2.3)$$

$$\hat{Y}_t = \phi \varepsilon_t^a + \phi \alpha \hat{K}_{t-1} + \frac{\phi \alpha}{\psi} \hat{r}_t^k + \phi(1 - \alpha) \hat{L}_t \quad (2.4)$$

where  $C_y$  and  $I_y$  are the steady state ratio of consumption and investment to output and  $\mathcal{M}$  is the monitoring costs faced by banks.  $\mathcal{M}$  is assumed to be negligible and is left out in the estimation process.  $\phi$  resembles a fixed cost of production and is assumed to be greater than 1.

The Linearized Taylor Equation that determines the nominal interest rate is

$$\hat{R}_t = \rho \hat{R}_{t-1} + (1 - \rho) \left[ r_{\pi_1} \hat{\pi}_t + r_{y_1} \hat{Y}_t + r_{\pi_2} \hat{\pi}_{t-1} + r_{y_2} \hat{Y}_{t-1} \right] + \varepsilon_t^r \quad (2.5)$$

The consumption and investment transition equations are

$$\hat{C}_t = \frac{h}{1+h} \hat{C}_{t-1} + \frac{1}{1+h} E_t[\hat{C}_{t+1}] - \frac{1-h}{(1+h)\sigma_c} \left( \hat{R}_t - E_t[\hat{\pi}_{t+1}] \right) + \varepsilon_t^b \quad (2.6)$$

$$\hat{I}_t = \frac{1}{1+\beta} \hat{I}_{t-1} + \frac{\beta}{1+\beta} E_t[\hat{I}_{t+1}] + \frac{1}{(1+\beta)S''} \hat{q}_t + \varepsilon_t^I \quad (2.7)$$

where  $\varepsilon_t^I$  and  $\varepsilon_t^b$  are exogenous stochastic stationary processes that effect the short term dynamics of consumption and investment.  $q_t$  is the relative price of capital and  $\beta$  is the discount rate.

The entrepreneurial return on capital is characterized by

$$\hat{R}_t^k - \hat{\pi}_t = \frac{1 - \tau}{1 - \tau + r^k} \hat{q}_t + \frac{r^k}{1 - \tau + r^k} \hat{r}_t^k - \hat{q}_{t-1} \quad (2.8)$$

The model yields a phillips curve equal to:

$$\hat{\pi}_t = \frac{\beta}{1 + \beta\iota_p} E_t[\hat{\pi}_{t+1}] + \frac{\iota_p}{1 + \beta\iota_p} \hat{\pi}_{t-1} + \frac{(1 - \beta\xi_p)(1 - \xi_p)}{(1 + \beta\iota_p)\xi_p} (\alpha\hat{r}_t^k + (1 - \alpha)\hat{w}_t - \hat{\varepsilon}_t^a) + \hat{\varepsilon}_t^p \quad (2.9)$$

where  $\xi_p$  is the degree of price stickiness,  $\iota_p$  is the degree of price indexation to last period's inflation rate and  $\hat{\varepsilon}_t^a$ ,  $\hat{\varepsilon}_t^p$  are exogenous processes that affect the productivity of production and the price mark up over marginal cost respectively.

Wages in the economy evolve according to:

$$\begin{aligned} \hat{w}_t = & \frac{\beta}{1 + \beta} E_t[\hat{w}_{t+1}] + \frac{1}{1 + \beta} \hat{w}_{t-1} + \frac{\beta}{1 + \beta} E_t[\hat{\pi}_{t+1}] - \frac{1 + \beta\iota_w}{1 + \beta} \hat{\pi}_t + \frac{\iota_w}{1 + \beta} \hat{\pi}_{t-1} \\ & - \frac{(1 - \beta\xi_w)(1 - \xi_w)}{(1 + \beta) \left(1 + \nu_l \frac{1 + \lambda_w}{\lambda_w}\right) \xi_w} \left( \hat{w}_t - \nu_l \hat{L}_t - \frac{\sigma_c}{1 - h} (\hat{C}_t - h\hat{C}_{t-1}) \right) + \hat{\varepsilon}_t^w \end{aligned} \quad (2.10)$$

where  $\xi_w$  is the degree of wage stickiness,  $\iota_w$  is the degree of wage indexation to last period's inflation rate and  $\hat{\varepsilon}_t^w$ , is an exogenous process that affect monopoly power households hold over labor.

The finance market is characterized by two equations, the first being the spread of the return on capital over the risk free rate:

$$\hat{S}_t \equiv E_t \left[ \hat{R}_{t+1}^k - \hat{R}_t \right] = \chi \left( \hat{q}_t + \hat{K}_t - \hat{n}_t \right) + \hat{\varepsilon}_t^F \quad (2.11)$$

where  $\chi$  is the elasticity of the spread with respect to the capital to net worth ratio and  $\hat{\varepsilon}_t^F$  is a finance shock that effects the riskiness of entrepreneurs and thus the riskiness of banks being paid back in full.

The second financial equation contains the evolutionary behavior of entrepreneur net worth:

$$\hat{n}_t = \delta_{\hat{R}^k}(\hat{R}_t^k - \hat{\pi}_t) - \delta_R(\hat{R}_{t-1} - \hat{\pi}_t) + \delta_{qK}(\hat{q}_{t-1} + \hat{K}_{t-1}) + \delta_n \hat{n}_{t-1} - \delta_\sigma \hat{\varepsilon}_{t-1}^F \quad (2.12)$$

where the  $\delta$  coefficients are functions of the steady state values of the loan default rate, entrepreneur survival rate, the steady state variance of the entrepreneurial risk shocks, the steady state level of revenue lost in bankruptcy, and the steady state ratio of capital to net worth. The value of  $\chi$ , which will be estimated, will determine the steady state level of the variance of the exogenous risk shock, the steady state value of the percentage of revenue lost in bankruptcy and the steady state level of leverage. Therefore, the value of  $\chi$  will determine the values of the  $\delta$  coefficients.<sup>1</sup>

In all, the SWFF model has eight exogenous shocks, seven of which are AR(1) processes the lone exception being the monetary policy shock which is simply white noise. All processes are assumed to be i.i.d. with mean zero and standard deviation  $\sigma_i$  and autocorrelation parameters  $\rho_i$ , where  $i = \{a, b, G, r, I, F, p, w\}$

## 2.3 SW Model

The SW model is identical to the SWFF model without the entrepreneur and banking sectors. Instead households own the capital, decide the utilization rate of capital, rent it to intermediate firms and sell it to capital producers. As a result the household budget constraint includes income received by renting and selling capital. In addition, households must choose how much capital to own.

The linearized first order condition of capital is given by

$$\hat{q}_t = -(\hat{R}_t - E_t[\hat{\pi}_{t+1}]) + \frac{1 - \tau}{1 - \tau + r^k} E_t[\hat{q}_{t+1}] + \frac{r^k}{1 - \tau + r^k} E_t[\hat{r}_{t+1}^k] + \hat{\varepsilon}_t^Q \quad (2.13)$$

This equation will replace the linearized equation (2.8). Since the equations (2.11) and (2.12)

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<sup>1</sup>For a comprehensive look at the functional forms of all the  $\delta$  coefficients used in coding the model, one must look at the working appendix of Del Negro and Schorfheide available at <http://economics.sas.upenn.edu/~schorf/research.htm>.

do not exist in the SW model there is a loss of an exogenous shock. In order to be able to directly compare misspecification error of the two models it is best that both models have the same amount of exogenous shocks. This is accomplished by adding a idiosyncratic equity premium price shock represented by  $\hat{\varepsilon}_t^Q$  to replace the finance shock  $\hat{\varepsilon}_t^F$  of the SWFF Model. Equation (2.13) is nested in the SWFF model if there exists no finance spread (i.e  $\hat{R}_{t+1}^k = R_t$ ). This assumption implies (2.8) forwarded ahead one period is identical to (2.13).

### 3 Estimation Technique

This section presents the steps needed to generate Bayesian estimates of the parameters of the linearized models of the previous section. For the Bayesian estimation, I adopt two techniques, the first being the standard Random Walk Metropolis-Hasting algorithm whose results will be referred to as SW-Reg and SWFF-Reg for the respective models. The second is a data-rich estimation method proposed by Boivin and Giannoni (2006) whose results will be referred to as SW-DFM and SWFF-DFM for the respective models. The Kalman filter is used to construct the likelihood of the models in both estimation techniques. Following Boivin and Giannoni (2006) and Kryshko (2011), I outline the steps of the Adaptive Metropolis-within-Gibbs algorithm used to estimate the SW-DFM and SWFF-DFM models. Next the priors for the models' parameters are shown and lastly, the data-set and its transformations are outlined in the final subsection.

#### 3.1 Regular DSGE Estimation

The state space representation of the solved model consists of a transition equation, which is calculated by solving the linearized system of the given model one wishes to evaluate for a given set of structural model parameters ( $\theta$ ):

$$S_t = G(\theta)S_{t-1} + H(\theta)v_t \quad \text{where } v_t \sim NID(0, I) \quad (3.1)$$

and the measurement equation:

$$X_t^{reg} = \Lambda S_t \quad (3.2)$$



Here  $X_t^{reg}$  are the economic data sets and  $\Lambda$  is a matrix matching the observed data to the definitions of the model's state variables  $S_t$ . The matrices  $G(\theta)$  and  $H(\theta)$  are functions of the model's structural parameters and  $v_t$  is a vector of the i.i.d. components of the model's exogenous processes  $\hat{\varepsilon}_t$ .

The description of the data sets and individual elements of  $\Lambda$  for the regular estimation technique can be found in Appendix A. With the model set up in state-space form and all stochastic processes being distributed normally and independently the Kalman Filter is used to calculate the likelihood function. Using the given priors found in Section 3.3, a Random-Walk Metropolis-Hastings<sup>2</sup> algorithm is then used to obtain the posterior distribution of the model's parameters  $P(\theta|X^{reg})$ .

### 3.2 DSGE-DFM Estimation

Bayesian estimation of a DSGE model in a data rich environment incorporates the state space model discussed above with a few modifications. The assumption that all relevant information for the estimation is summarized by a relatively small number of data sets needs to be met in order for accurate estimates and forecasts to be obtained when a DSGE model is estimated as described in Section 3.1. However, the development of Dynamic Factor Models proposed by Sargent and Sims (1977) and further advanced by the works of Stock and Watson (1989, 2003, 2005, 2009) have shown that large data sets can hold valuable information in identifying unobserved common factors of the economy.

Further, the abundance of data series that can stand in as a measurable metric of a particular economic variable can be large as well, for example, inflation can be measured in multiple data sets including CPI, PCE, GDP deflator and other series. The econometrician's choice of which data set(s) to use in the estimation process can have an impact on the results as shown by Guerron-Quintana (2010).

The state space set up for DSGE-DFM estimation is characterized by equations (3.3)-(3.5).

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<sup>2</sup>For more detail on Bayesian DSGE estimation techniques please see An and Schorfheide (2007)

$$S_t = G(\theta)S_{t-1} + H(\theta)v_t \text{ where } v_t \sim NID(0, I_m) \quad (3.3)$$

$$X_t = \Lambda S_t + e_t \quad (3.4)$$

$$e_t = \Psi e_{t-1} + \epsilon_t \text{ where } \epsilon_t \sim NID(0, R) \quad (3.5)$$

Here  $e_t$  follows an AR(1) process and is often referred to as measurement error. The matrix  $X$  is  $J \times T$  where  $J$  is the number of data series used in estimation and  $T$  is the number of observables for each series. The Matrix  $\Lambda$  is now no longer assumed to be known by the econometrician, but instead is estimated within the MCMC routine. The matrices  $\Psi$  and  $R$  that govern the measurement error's stationary processes for each series are assumed to be diagonal and are also estimated within the MCMC routine.

The measurement equation (3.4) has the following structure:

$$\begin{bmatrix} \text{Output \#1} \\ \text{Output \#2} \\ \text{Inflation \#1} \\ \text{Inflation \#2} \\ \vdots \\ \text{-----} \\ [\text{Housing Market}] \\ [\text{Labor Market}] \\ [\text{Output Components}] \\ [\text{Financial Market}] \\ [\text{Investment}] \\ [\text{Price/Wage Indexes}] \\ [\text{Other}] \end{bmatrix} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ \lambda_{Y_1} & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ 0 & \lambda_{\pi_2} & \dots & 0 \\ \text{---} & \text{---} & \text{---} & \text{---} \\ [\lambda_{H_1}] & [\lambda_{H_2}] & \dots & [\lambda_{H_n}] \\ [\lambda_{L_1}] & [\lambda_{L_2}] & \dots & [\lambda_{L_n}] \\ \vdots & \vdots & \dots & \vdots \end{bmatrix} \begin{bmatrix} \hat{Y}_t \\ \hat{\pi}_t \\ \vdots \\ \epsilon_t^f \end{bmatrix} + \begin{bmatrix} e_t \end{bmatrix}$$

where  $X_t$  is partitioned into core series and non-core series separated by the dashed line. The core series are series that are only allowed to load on one particular variable of the state vector  $S_t$  to which there is a known sole relationship between series and state. (For

instance, GDP to  $Y$ ) Further, the factor loading coefficient for the first series of each core variable that corresponds to a particular known state is assumed to be perfectly tight, this is represented by the 1's in the  $\Lambda$  matrix. This anchors the estimated states of the DSGE model and ensures that they don't drift too far away from their theoretical foundation.

The non-core series consist of the remaining 97 data sets not in the core series and are grouped into eight subgroups. These series are allowed to 'load' on all time  $t$  states in the state vector. Non-core series may have up to  $n$  (where  $n$  is the number of elements in  $S_t$ ) non-zero elements for their corresponding row in  $\Lambda$  unlike the core series whose corresponding row in  $\Lambda$  may only have one non-zero element.

Following the work of Boivin and Giannoni (2006) and Kryshko (2011) a Metropolis-within-Gibbs algorithm is used to estimate the state space parameters  $\Gamma = [\Lambda, \Psi, R]$  and the structural DSGE parameters  $\theta$ . The likelihood functions of the DSGE-DFM models appear to have many peaks and cliffs that can cause the MCMC algorithm to get "stuck" in places. To make sure the algorithm explores the entirety of the parameter space, I have implemented an adaptive element into the Metropolis step of the algorithm along the lines of Roberts and Rosenthal's (2009) adaptive within Gibbs example. The adaptive Metropolis-within-Gibbs algorithm used follows the following steps:

1. Specify Initial values of  $\theta^{(0)}$ , and  $\Gamma^{(0)}$ ,  $\Gamma = \{\Lambda, \Psi, R\}$
2. Repeat for  $g=1 \dots G$ 
  - 2.1 Solve the DSGE model numerically and obtain  $G(\theta^{(g-1)})$  and  $H(\theta^{(g-1)})$
  - 2.2 Draw from  $p(\Gamma|G(\theta^{(g-1)}), H(\theta^{(g-1)}); X_{1:T})$ 
    - 2.2.1 Generate unobserved states  $S^{1:T,(g)}$  from  $p(S^T|\Gamma^{(g-1)}, G(\theta^{(g-1)}), H(\theta^{(g-1)}); X_{1:T})$  using the Carter-Kohn forward-backward algorithm
    - 2.2.2 Generate state-space parameters  $\Gamma^{(g)}$  from  $p(\Gamma|S^{1:T,(g)}; X_{1:T})$  by drawing from a set of known conditional densities  $[R|\Lambda, \Psi; S^{1:T,(g)}]$ ,  $[\Lambda|R, \Psi; S^{1:T,(g)}]$ ,  $[\Psi|\Lambda, R; S^{1:T,(g)}]$ .
  - 2.3 Draw DSGE parameters  $\theta^{(g)}$  from  $p(\theta|\Gamma; X_{1:T})$  using adaptive Metropolis Hastings
    - 2.3.1 Propose  $\theta^* = \theta^{(g-1)} + \bar{c}\varepsilon_\ell$  where  $\varepsilon_\ell \sim NID(0, \Sigma^{-1})$
    - 2.3.2 Calculate  $P(X_{1:T}|\theta^*, \Gamma^{(g)})$  using the Kalman Filter

2.3.3 Calculate the acceptance probability  $\omega$

$$\omega = \min \left\{ \frac{P(X_{1:T}|\theta^*, \Gamma^{(g)})P(\theta^*)}{P(X_{1:T}|\theta^{(g-1)}, \Gamma^{(g)})P(\theta^{(g-1)})}, 1 \right\}$$

2.3.4  $\theta^{(g)} = \theta^*$  with probability  $\omega$  and  $\theta^{(g)} = \theta^{(g-1)}$  with probability  $(1 - \omega)$

2.4 Calculate acceptance rate of proposed  $\theta$  for 1 to  $g$  draws. If the acceptance rate is lower than target acceptance rate decrease  $\bar{c}$  by  $w$  (iff  $\bar{c} > w$ ), if acceptance rate is greater than target acceptance rate increase  $\bar{c}$  by  $w$ . This target acceptance rate adaption can be implemented every  $n$  iterations of  $g$ . In addition the condition  $w \rightarrow 0$  as  $g \rightarrow \infty$  must be satisfied

3. Return  $\{\theta^{(g)}, \Gamma^{(g)}\}_{g=1}^G$

A few comments are in order. First, regarding step 2.2 which is the Gibbs portion of the algorithm. This step uses the Carter-Kohn (1994) algorithm which first requires a forward pass of the Kalman filter to collect the generated states,  $S$ , and their corresponding cov/var matrices,  $P$ . The backward pass of the algorithm then smooths out the estimated states using both  $S$  and  $P$  from the forward pass.<sup>3</sup> Step 2.2.2 then performs line-by-line OLS for each series in  $X$  given the generated states  $S^{1:T}$ . With the use of the proper conjugate priors the distributions of step 2.2.2 are known using the approach of Chib and Greenberg (1994).

The algorithm must first be initialized with  $\theta^{(0)}$ ,  $\Gamma^{(0)}$  and  $\Sigma$ . The values of  $\theta^{(0)}$  are retrieved by taking the mean of  $P(\theta|X^{reg})$  when estimated as described in Section 3.1. Once  $\theta^{(0)}$  is obtained it is then used to calculate  $S^{1:T,(0)}$ . The estimated states are then used to run line-by-line OLS for each series in  $X$  to back out initial values of  $\Gamma^{(0)}$ .  $\Sigma^{-1}$  is the inverse Hessian of the DSGE model evaluated at its posterior mode under regular estimation.

The applied algorithm is based on 500,000 draws (2 parallel chains of 250,000 draws discarding the initial burn-in period of 100,000 iterations). The calibrations regarding the adaptive step include the acceptance target rate which is set at 27%, an initial  $\bar{c}$  which is set

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<sup>3</sup>The backwards pass draws states  $S$  using a cov/var matrix that is a transformation of the  $P$  matrix. It is necessary that  $P$  be a symmetric and positive semi-definite matrix. However, it is sometimes necessary to computationally transform the  $P$  matrix using the procedure outlined by Rebonato (1999)

to .1, the adaptive jump size  $w$  which is set at  $.005^4$  and an adjustment rate  $n$  which is set at 25. The adjustment rate  $n$  determines how many iterations take place between changing  $\bar{c}$  as described in step 2.4.

### 3.3 Data and Parameter Priors

To estimate both the SW and SWFF models in a data-rich environment a total of 97 quarterly<sup>5</sup> data series are used. These series cover the time period of 1984Q2 to 2008Q3. The complete set of series encompasses many of the economic and financial series used by Stock and Watson (2009) and Kryshko (2011). The evaluation window of the data series is significant for multiple reasons. First, Kim and Nelson (1999) have argued that a structural break in economic growth volatility occurred in 1984Q1. Clarida et al. (2000) have shown that the stability of monetary policy of the form of equation (2.5) did not occur until the early 1980's. Further, Lubik and Schorfheide (2004) assert that it was not until the early 1980's that monetary policy of this form was consistent with a determinate equilibrium. Finally, 2008Q3 was the last quarter before nominal interest rates hit the zero lower bound.

The SWFF-DFM (SW-DFM) estimation consist of 17 (15) core series and 80 (82) non-core series. The core series for both models include three measures each of GDP, inflation, employment and nominal interest rates. Also included in the core series are real consumption and investment expenditures and hourly wages. In addition, the core series for the SWFF-DFM model include 2 measures of the interest rate spread. The series that hold a perfectly tight loading factor are the 8 (7) series used in regular estimation of each model. These include real per capita GDP, the GDP price deflator, per capita real consumption and private investment expenditures, real average hourly wage, hours worked, the annualized federal funds rate and the quarterly spread between BAA corporate bond yields to the 10 year Treasury bond yield. All per capita variables are calculated using the adult population of 16 years and older. These series are either demeaned, linearly detrended log level or log first differenced and demeaned<sup>6</sup>. A complete list and transformation rubric of each core

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<sup>4</sup>In order to accord with the condition of step 2.4,  $w = \min\left(.005, \left(\frac{g}{n}\right)^{-.5}\right)$

<sup>5</sup>A 3-month average is used to obtain quarterly data from monthly series

<sup>6</sup>This is to account for no intercept vector in the measurement equation

series along with their corresponding Fred-II database code is found in Appendix A.

The non-core series are grouped into eight categories. The first being *Output Components* which include series that explain deviations from per capita linear trends of different GDP and production output components. The *Labor Market* category includes employment by sector as well as unemployment rates and durations. The *Housing Market* group includes regional housing starts and the residential investment series. The *Financial Market* classification includes a number of different interest rates, loan and credit quantities and asset prices. The *Exchange Rate* group includes exchange rates of the US dollar to other foreign currencies. The *Investment* grouping includes inventory indexes and other investment series. The *Price and Wage* category includes a number of pricing indicies, wage indicies and commodity prices. The final category *Other* includes money supply measures and consumer and producer sentiment surveys.

As is common in the Dynamic Factor Model literature, all non-core series sample standard deviation is normalized to 1. In addition, these series are either demeaned, linearly detrended log level or log first differenced and demeaned. A complete list and transformation rubric of each non-core series along with their corresponding Fred-II database code is found in Appendix A.

The structural parameter marginal priors are in accordance to the Smets and Wouters (2003, 2007) priors. The parameter priors include normal, beta, gamma, and inverse gamma distributions. All coefficients whose values lie within the unit interval are drawn from beta distributions, while all standard deviations of the structural shocks are drawn from inverse gamma distributions. The priors on the autocorrelation coefficients of the structural shock ensure that shocks will be persistent in the model economy. The joint prior is given by the products of the marginals and is truncated to parameter values that guarantee a determinate and unique model equilibrium. The distribution of the prior along with its mean, standard deviation and description of the parameter are laid out in Table 1.

In addition, some structural parameters are fixed including the discount rate, share of capital, depreciation rate, and the steady state share of government and investment to total output. The latter parameters being calibrated to the average proportion of investment and government purchases of GDP over the sample period. In the SWFF model the steady

**Table 1:** Priors for DSGE Models' Parameters

|                                       | Description                          | Distribution | Mean  | Std   |
|---------------------------------------|--------------------------------------|--------------|-------|-------|
| <b>Structural Parameters</b>          |                                      |              |       |       |
| $\psi$                                | Capital utilization costs            | Beta         | 0.2   | 0.08  |
| $\iota_p$                             | Degree of indexation on prices       | Beta         | 0.5   | 0.15  |
| $\iota_w$                             | Degree of indexation on wages        | Beta         | 0.5   | 0.15  |
| $\xi_p$                               | Calvo price stickiness               | Beta         | 0.6   | 0.05  |
| $\xi_w$                               | Calvo wage stickiness                | Beta         | 0.6   | 0.05  |
| $\nu_l$                               | CRRA coef. on labor                  | Gamma        | 1.4   | 0.45  |
| $\sigma_c$                            | CRRA coef. on consumption            | Gamma        | 1.2   | 0.45  |
| $h$                                   | Habit consumption                    | Beta         | 0.7   | 0.1   |
| $\phi$                                | Fixed cost of production             | Gamma        | 0.5   | 0.3   |
| $S''$                                 | Capital adjustment cost              | Normal       | 5     | 1     |
| <b>Policy Parameters</b>              |                                      |              |       |       |
| $r_{\pi_1}$                           | Taylor Rule coef. on inflation       | Gamma        | 2     | 0.33  |
| $r_{y_1}$                             | Taylor Rule coef. on output gap      | Gamma        | 0.2   | 0.1   |
| $r_{\pi_2}$                           | Taylor Rule coef. on past inflation  | Normal       | -0.3  | 0.1   |
| $r_{y_2}$                             | Taylor Rule coef. on past output gap | Normal       | -0.06 | 0.05  |
| $\rho$                                | Lagged interest rate in Taylor Rule  | Beta         | 0.7   | 0.1   |
| <b>Exogenous Processes Parameters</b> |                                      |              |       |       |
| $\rho_a$                              | AR(1) coef. on productivity shock    | Beta         | 0.8   | 0.1   |
| $\rho_b$                              | AR(1) coef. on preference shock      | Beta         | 0.8   | 0.1   |
| $\rho_G$                              | AR(1) coef. on gov't spending shock  | Beta         | 0.8   | 0.1   |
| $\rho_I$                              | AR(1) coef. on investment shock      | Beta         | 0.8   | 0.1   |
| $\rho_w$                              | AR(1) coef. on wage mark-up shock    | Beta         | 0.5   | 0.1   |
| $\rho_p$                              | AR(1) coef. on price mark-up shock   | Beta         | 0.5   | 0.1   |
| $\sigma_a$                            | Std. of productivity shock           | Inv. Gamma   | 0.1   | 2*    |
| $\sigma_b$                            | Std. of preference shock             | Inv. Gamma   | 0.1   | 2*    |
| $\sigma_G$                            | Std. of gov't spending shock         | Inv. Gamma   | 0.1   | 2*    |
| $\sigma_r$                            | Std. of monetary policy shock        | Inv. Gamma   | 0.1   | 2*    |
| $\sigma_I$                            | Std. of investment shock             | Inv. Gamma   | 0.1   | 2*    |
| $\sigma_p$                            | Std. of price mark-up shock          | Inv. Gamma   | 0.1   | 2*    |
| $\sigma_w$                            | Std. of wage mark-up shock           | Inv. Gamma   | 0.1   | 2*    |
| $\sigma_q$                            | Std. of equity premium shock         | Inv. Gamma   | 0.1   | 2*    |
| <b>Parameters Specific to SWFF</b>    |                                      |              |       |       |
| $\chi^*$                              | Spread Elasticity                    | Beta         | 0.05  | 0.005 |
| $\rho_F$                              | AR(1) coef. on finance shock         | Beta         | 0.8   | 0.1   |
| $\sigma_F$                            | Std. of finance shock                | Inv. Gamma   | 0.1   | 2*    |

Note: the auxiliary parameter  $\chi$  is estimated with  $\chi^* = .0225 + .0825\chi$

Note: All inverse gamma distributions list degrees of freedom instead of std.

state default rate is set to .0075 which corresponds to Bernanke, Gertler, Gilchrist (1999) annualized default rate of 3%. The quarterly survival rate of entrepreneurs is fixed at .99 which corresponds to an average entrepreneur life of 68 quarters or 17 years. The steady state spread is calibrated to 140 basis points which is roughly the sample median spread between the BAA corporate bond yield and 10 year Treasury bond. yield. This value is in line with the estimated values of Del Negro et al. (2013) who estimated the steady state spread to be between 73 and 150 basis points. A complete list of calibrated structural parameters can be found in Table 2.

**Table 2:** Calibrated Parameters

|                         | Description                         | Value  |
|-------------------------|-------------------------------------|--------|
| $\beta$                 | Discount rate                       | 0.99   |
| $\alpha$                | Share of capital                    | 0.3    |
| $\tau$                  | Depreciation rate                   | 0.025  |
| $I_y$                   | S.S investment proportion of output | 0.18   |
| $g_y$                   | S.S government proportion of output | 0.19   |
| $\lambda_w$             | Degree of wage markup               | 0.3    |
| <b>Specific to SWFF</b> |                                     |        |
| $\gamma$                | Survival rate of entrepreneur       | 0.99   |
| $F^*$                   | Loan default rate                   | 0.0075 |
| $S$                     | S.S. Spread (Annual %)              | 1.4    |

The priors for the state space parameters include the elements of  $\Lambda$  and the diagonal elements of  $\Psi$  and  $R$ . First, the elements of  $\Lambda$  can be separated between core and non-core elements. Core series may only have a single non-zero row element of  $\Lambda$  whose prior is normally distributed and centered around 1<sup>7</sup>. Each non-core series corresponding row elements<sup>8</sup> of  $\Lambda$  has a multivariate normal prior centered around zero.

The prior for each  $i^{th}$  row of the non-core series follows the work of Boivin and Giannoni (2006) and Kryshko (2011), who use a Normal-Inverse-Gamma prior distribution for  $(\Lambda_i, R_{i,i})$  so that  $R_{i,i} \sim IG_2(.001, 3)$  and the prior mean of factor loadings for the  $i^{th}$  row is given by  $\Lambda_i | R_{i,i} \sim N(0, R_{i,i}I)$  where the mean is a vector of zeros and  $I$  is the identity matrix. The

<sup>7</sup>The core interest rate series priors are centered around 4 since the interest rates are in annualized percentage

<sup>8</sup>The elements of  $\Lambda$  that correspond to  $t - 1$  states of the  $S_t$  vector are assumed to be zero



prior for the  $i^{th}$  measurement equation's autocorrelation parameter,  $\Psi_{i,i}$  is  $N(0, 1)$  for all rows. The autocorrelation parameter prior is truncated to values inside the unit circle to ensure all error processes are stationary.

Priors regarding the core series is still Normal-Inverse-Gamma but instead the mean of the factor loadings of the  $i^{th}$  row of  $\Lambda$  is centered at the DSGE models implied theoretical loading. As discussed earlier the first data set of each core series category has a perfectly tight loading prior. The priors for  $\Psi$  and  $R$  whose diagonal elements correspond to core series remains the same. In the spirit of Boivin and Giannoni (2006) who fix the measurement equation of the federal funds rate error term to be zero and Kryshko (2011) who fixes all Taylor Rule policy parameters to be equal to the means of the posterior distributions estimated in the regular environment, I truncate  $R_{13,13}$  which correspond to the federal funds rate error term to be no greater than 0.05. This assures that the nominal interest rate of the DSGE model will not drift far away from the federal funds rate observed in the economy.

**Table 3:** Priors for DSGE-DFM  $\Gamma$  Parameters

|                                       | Description                           | Distribution | Mean  | Std        |
|---------------------------------------|---------------------------------------|--------------|-------|------------|
| <b><math>\Gamma</math> Parameters</b> |                                       |              |       |            |
| $\Psi_{i,i}$                          | AR(1) coef. of misspecification error | Normal       | 0     | 1          |
| $R_{i,i}$                             | Variance of misspecification error    | Inv. Gamma   | 0.001 | 3*         |
| $\Lambda_{i,j}$                       | Factor loadings of Non-core data sets | Normal       | 0     | $R_{i,i}I$ |
| $\Lambda_{i,j}$                       | Factor loadings of Core data sets     | Normal       | 1     | $R_{i,i}I$ |

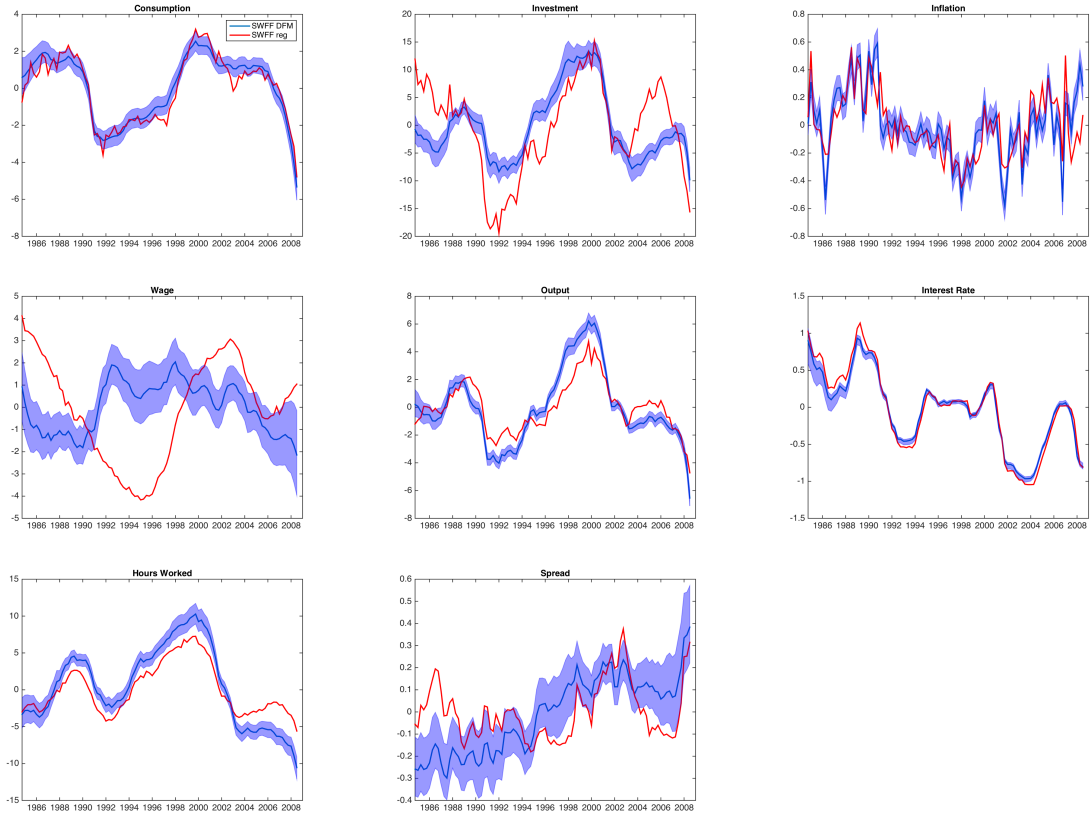
## 4 Dynamics of the SWFF-DFM Model

In this section, I illustrate some of the key economic mechanisms at work in SWFF-DFM model. I do so with the aid of impulse response functions. The posterior estimates for the structural parameters for the SWFF-Reg and SWFF-DFM models are tabulated in Table 4. I save discussion of these parameter estimates until Section 5.1.

## 4.1 Estimated State Variables

Before looking at the dynamics generated by shocks inside the SWFF-DFM model lets first look at the in-sample dynamics of the model to ensure that our SWFF-DFM model is consistent (in terms of the conduct of macroeconomic series) with the in-sample dynamics of the SWFF-Reg model. Using the Carter-Kohn algorithm which is applied in the DSGE-DFM estimation algorithm it is straightforward to calculate the estimates of the endogenous and exogenous variables of the model over the sample time period. These are plotted for the SWFF model in Figures 1 to 3. The blue line and shaded area represent the posterior mean and 90% density interval of the variable under SWFF-DFM estimation and the red line and shaded area represent the posterior mean and 90% density interval of the variable under SWFF-Reg estimation. The y-axis of all plots is representing percentage deviations away from the variables steady state values.

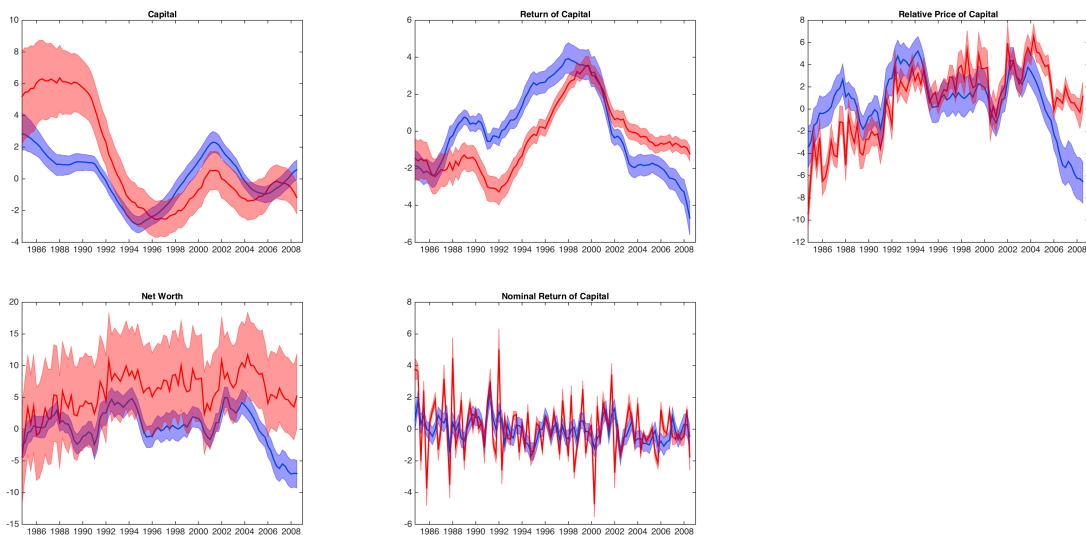
**Figure 1:** Simulated States of Endogenous Variables of SWFF



The eight plots of Figure 1 represent endogenous variables that are directly related to a data series in the core.<sup>9</sup> Recall, one of the series has a perfectly tight loading prior to ensure that the variable is “anchored” to its economic definition. As the plots show this is indeed the case, with the first eight endogenous variables within the same neighborhood of the SWFF-Reg endogenous variable estimations.

The five plots of Figure 2 and the eight plots of Figure 3 correspond to variables not directly linked to a particular class of economic variable. As a result, the percent deviations from steady state of the time plots of these variables exhibit noteworthy differences between the estimation techniques. These variables are where the large data set can most easily load and generate dynamic state factors to help explain the large set of data while still possessing the theoretical structure inside the DSGE model. Also of note is that these variables exhibit smoother and smaller posterior density intervals when estimated in the data-rich environment.

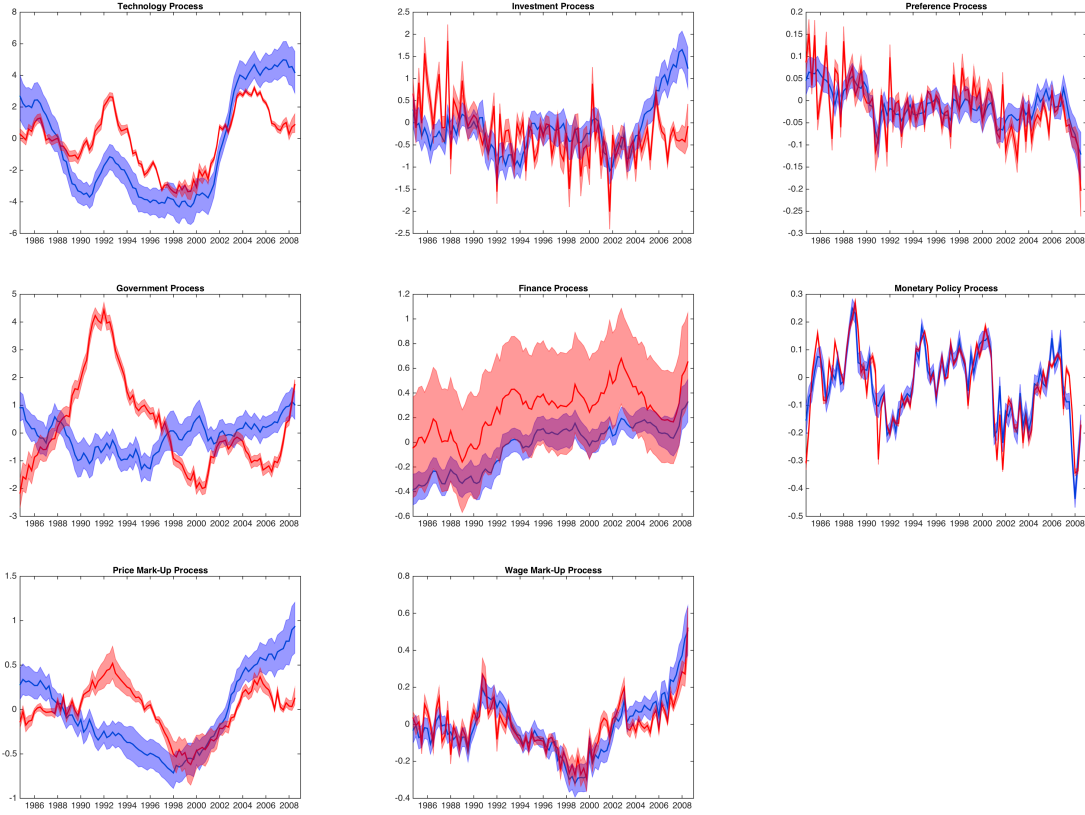
**Figure 2:** Simulated States of Latent Endogenous Variables of SWFF




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<sup>9</sup>Since, there is assumed to be no measurement/misspecification error in the SWFF-Reg estimation there is no posterior density interval around the first eight endogenous variables as they are assumed to be measured without error.

**Figure 3:** Simulated States of Exogenous Processes of SWFF



## 4.2 Impulse Response Functions

DSGE-DFM estimation allows for economic series not directly corresponding to any endogenous variables in the DSGE model to be related to the model's exogenous shocks. This allows IRF's to be generated for many economic series whose IRF's do not exist outside of structural VAR estimation. This can also act as a rudimentary diagnostic tool of how well the DSGE model is identified and specified. For example, if it is found that many of the price indexes included in the data set fall when there is a positive price shock it would tend to suggest an identification problem. Figures 4 to 7 represent such IRF's for the SWFF-DFM model and are discussed in this subsection.

Figure 4 gives the IRF's and 80% posterior density band of a one unit negative finance shock (positive spread shock). The red IRF's correspond to the same one unit finance shock but are only available for series used in DSGE-Reg estimation. Although all shocks are unitary the estimated standard deviation for the shock can differ. The IRF's shows that the finance shock lowers Real GDP and increases the spread as the finance accelerator would predict. Notice that the impact on the spread is smaller in the SWFF-DFM model but its impact is larger on Real GDP when compared to the SWFF-Reg model. This is due to the higher estimate of the spread elasticity in the financial accelerator in the SWFF-DFM model. In addition, the unemployment rate increases and peaks about 7 quarters after the shock and results in a longer average unemployment duration in the future. The adverse finance shock results in the decrease of manufacturing employees captured by the 5th plot of the diagram and commercial loans begin to fall a few quarters after the finance shock. As the SWFF model theoretically predicts, a finance shock increases entrepreneurial risk and investment loan quantities decrease inside the SWFF model.

**Figure 4: IRF's of Negative Finance Shock**

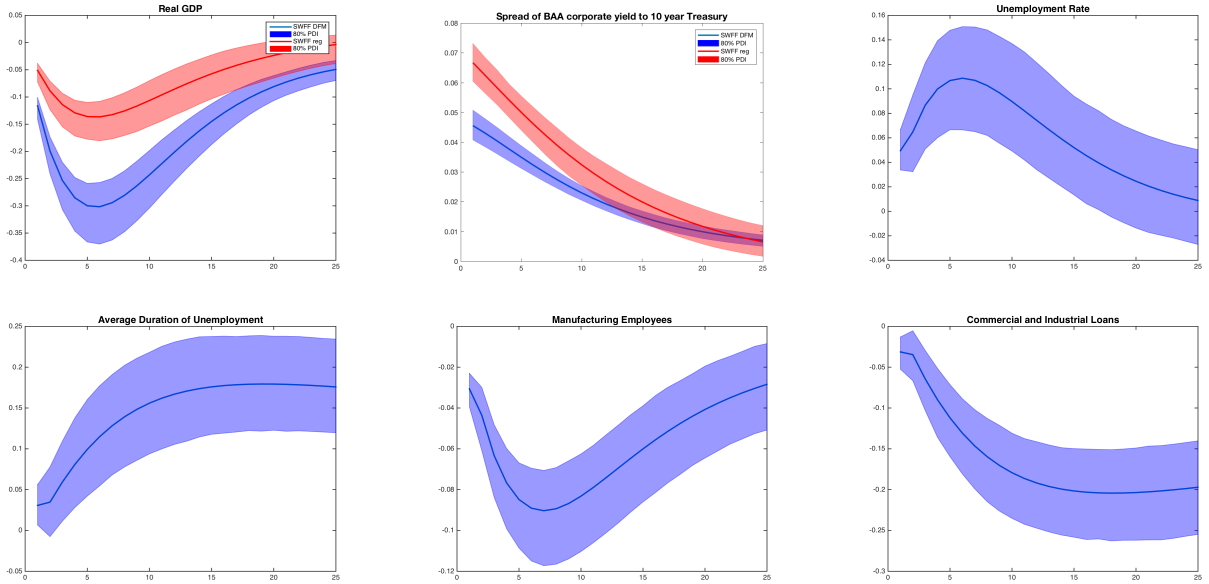
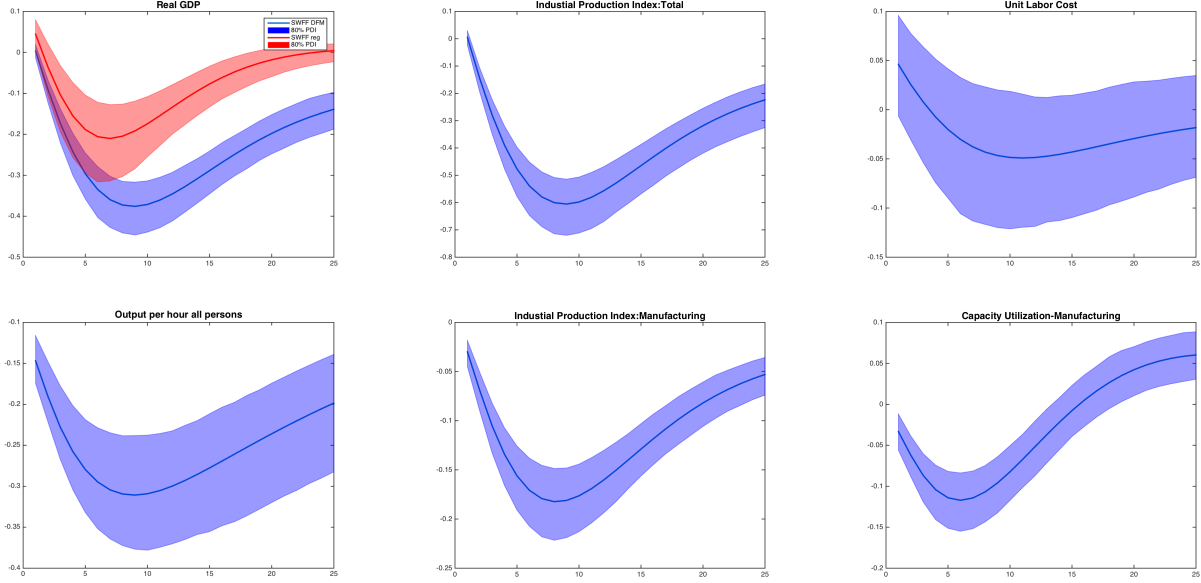


Figure 5 gives the IRF's of a negative productivity shock in the economy. It depicts that Real GDP and Industrial Production Indexes all fall and are hump shaped with a trough around 6-8 quarters. Capacity Utilization in the manufacturing sector falls and output per

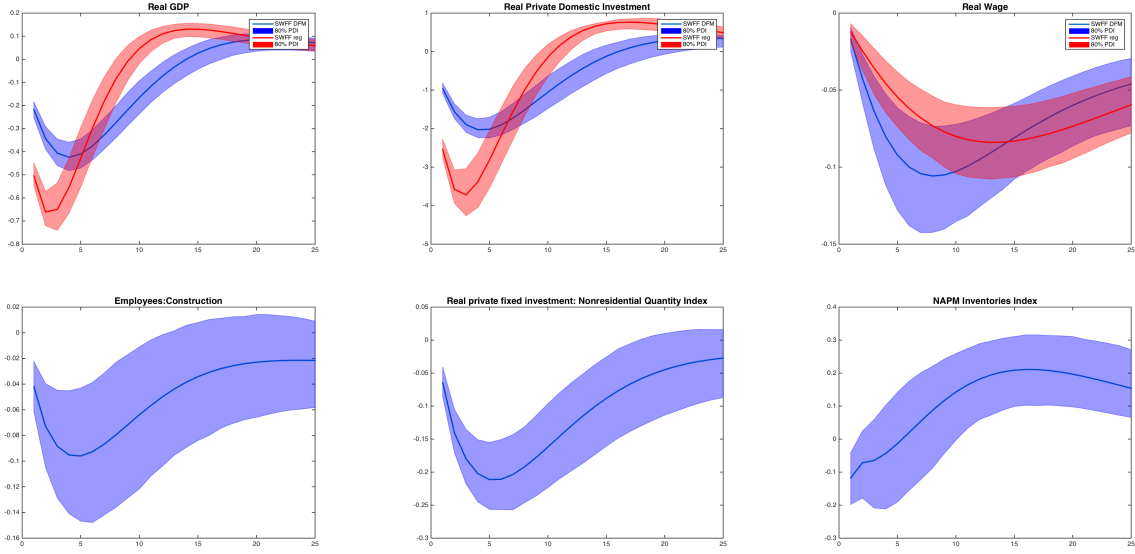
hour of all persons falls as well. This is of particular note as it is the closest data series I have that could be thought of as a measurable for labor productivity.

**Figure 5: IRF's of Negative Productivity Shock**



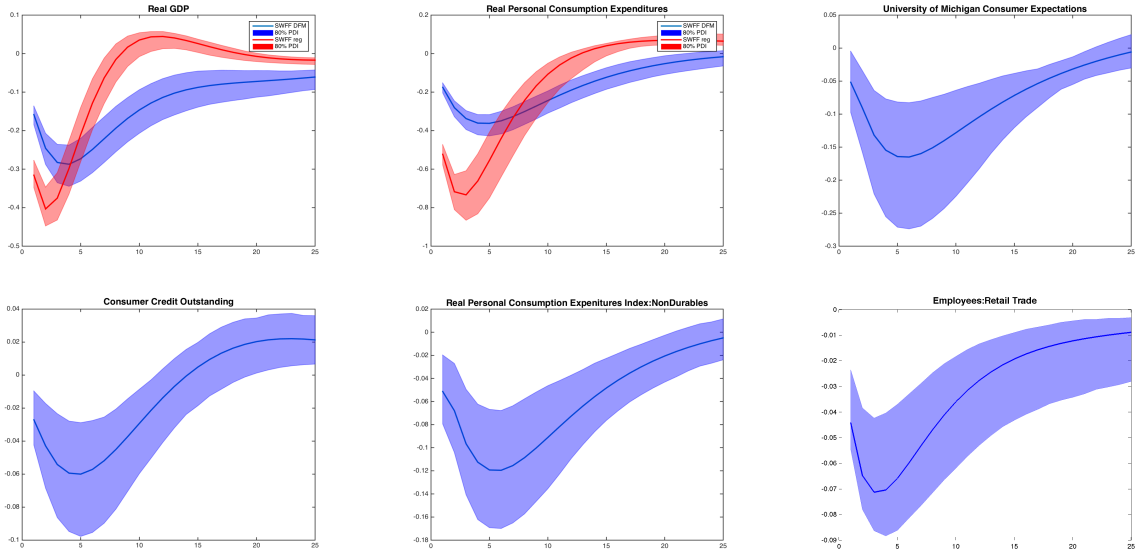
The next set of IRF's plot a negative investment shock in Figure 6. As expected real investment falls in both the SWFF-Reg model and SWFF-DFM model. However, the degree to which they fall and how fast they recover is quite substantial. This is due to the smaller estimates of the average size of an investment shock in the SWFF-DFM model. There is also a decrease in non-residential investment, business inventories and new orders. The pro-cyclical relationship seen in the data between real wages and real GDP remains consistent inside the model. Inventories initially decrease, but as the economy begins to recover, inventories begin to exceed their long-run averages about 8-12 quarters after the investment shock.

**Figure 6: IRF's of Negative Investment Shock**



The final set of IRF's are plotted in Figure 7 and are associated with a negative preference shock (negative consumption shock). The IRF's show a downturn in real GDP, real personal consumption and consumption expenditures on non-durables. Such a shock corresponds to a decrease in employees in the retail sector and a decrease in outstanding consumer credit. Interestingly, the negative preference shock also corresponds to a decrease in the University of Michigan's Consumer Expectations Survey.

**Figure 7: IRF's of Negative Preference Shock**



### 4.3 Comparing the Economic Effects of Normalized Structural Shocks

The DSGE-DFM framework can help in answering questions like: what makes financial recessions and subsequent recoveries so much different than other recessions and recoveries? I attempt to evaluate such a question by comparing the IRF's of different normalized structural shocks. All the negative structural shocks discussed in the previous subsection decrease output. A closer examination of related macroeconomic series show that these structural shocks are theory-consistent with series directly linked inside the model and series indirectly linked to the model. The SWFF-DFM model displays that the greatest and most persistent decreases in output are associated with negative financial and productivity shocks. Yet, these shocks and their resulting dynamics do not account for different magnitudes of decrease in output between the different shocks. In order to trace the dynamic effects of the structural shocks to additional data indicators I must normalize the structural shocks to assure that output falls by a similar magnitude across the menu of structural shocks.

To conduct this application, I calibrate all parameters including the loading coefficients of the SWFF-DFM model to their estimated posterior median and normalize the size of the eight structural shocks to ensure that the maximum decrease of real output is equal across the different shocks.<sup>10</sup> This assures any differences in the fluctuations of other variables or series are not due to an output level effect. Figures 8 and 9 examines the IRF's of each structural shock for nine different economic series. Since the paper is mainly focused on financially driven recessions, I highlight the IRF's equated to the financial shock by the thick green line in Figure 8 and 9. Further, unlike the financial shocks seen in 2008, none of these negative shocks create a deep enough decline in output to force the model below the zero lower bound.

Figure 8 plots the IRF's of real GDP, Investment, Exports and Residential Investment. Notice that by design real GDP decreases by the same amount for each of the structural shocks. However, notice that this decrease in GDP is quickest after negative monetary and

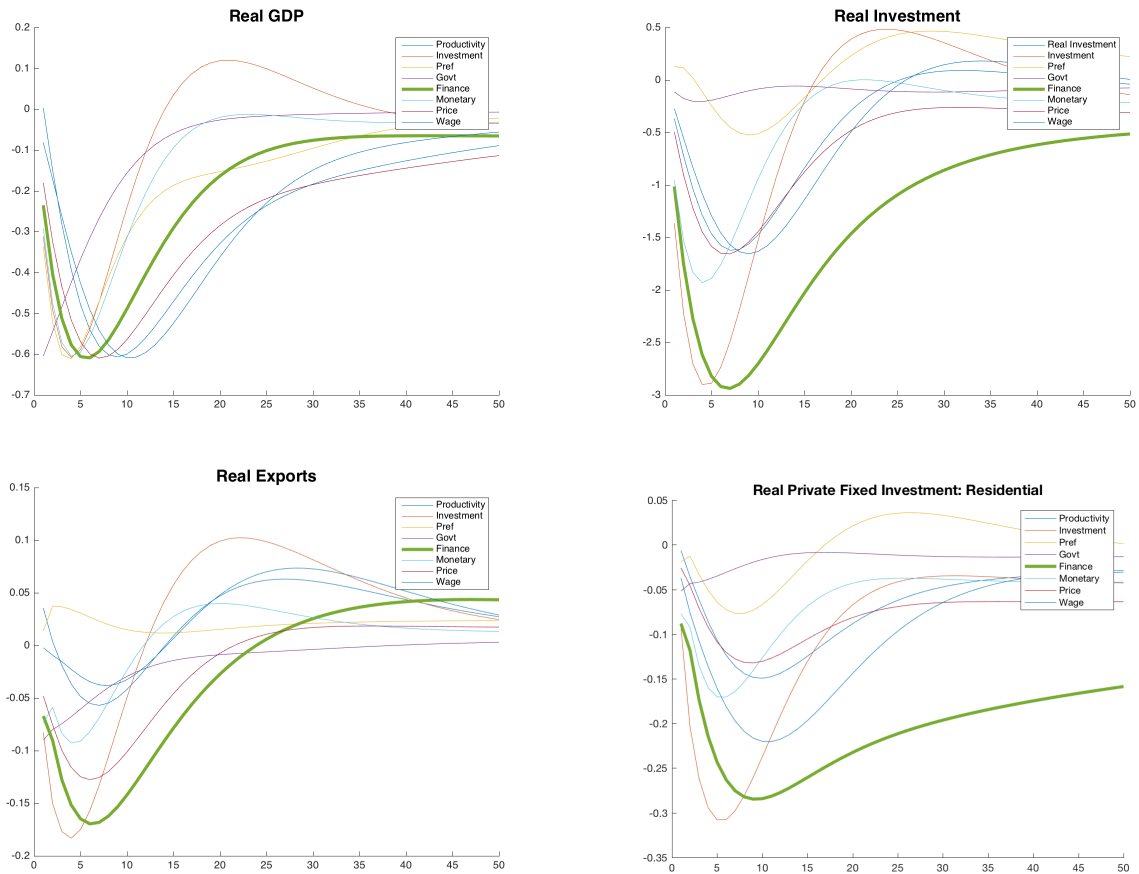
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<sup>10</sup>The decrease of real output is normalized around the decrease associated with a two standard deviation financial spread shock.



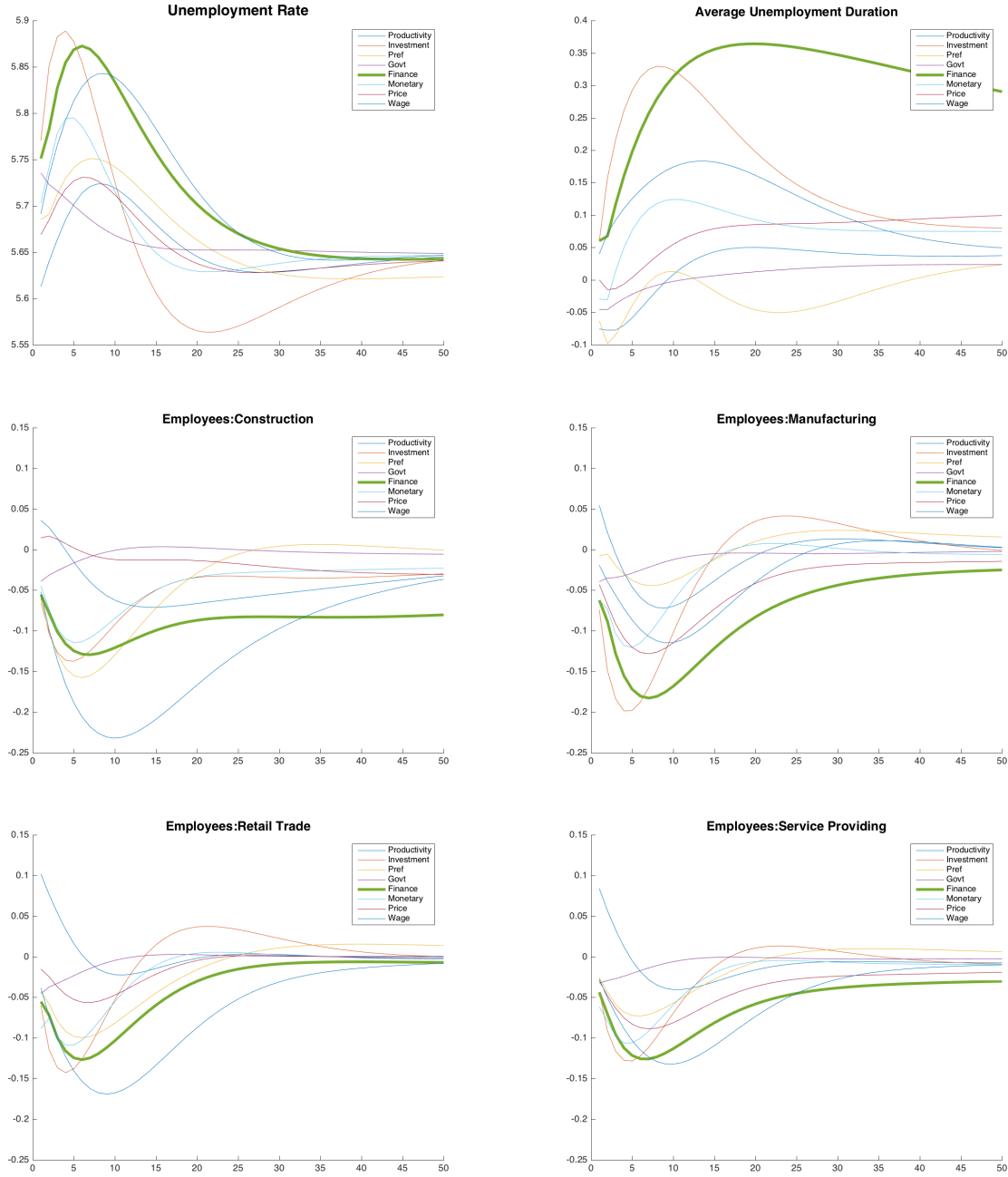
consumer shocks, as recovery starts 4-5 quarters after the shock. Recoveries after negative investment and financial shocks start 5-6 quarters after the shock, while recoveries after negative supply shocks (productivity and wage shocks) have more persistence, as they do not begin until 7 or 8 quarters after the initial shock. I also see that particular components of GDP react much differently to what has caused the decline in output. Real Investment and real Exports decrease by a much larger amount and are slower to recover to their steady state value after a financial shock. The decreases in both is similar to that of a negative investment productivity shock but recover at a much slower pace. Recovery for real Residential Investment remains extremely sluggish after a negative financial shock compared to any other type of shock.

**Figure 8: Comparing Normalized IRF's**



When I study the IRF's for different labor market measures, including the unemployment rate and average unemployment duration time, I observe that the effect on both differ

**Figure 9:** Comparing Normalized IRF's



depending on what mechanism is behind the output decline. Since the decrease in real GDP is identical, the different unemployment rate dynamics would suggest that the coefficient on Okun's Law is different depending on what the driving force behind the decrease in output is. The unemployment rate increase is largest after negative investment and financial shocks,

but the inertia associated with financial shocks is much greater, as the unemployment rate and the average duration of unemployment remains high for much longer when compared to any other type of shock.

If I examine the labor market in closer detail it sheds light on why this phenomenon of a high and persistent unemployment rate may occur. I see that the decrease in inventories and real investment are largest and most persistence after a financial shock. As a result the number of employees in manufacturing and construction decreases most significantly after financial shocks, while the decreases of service providing and retail trade jobs after a financial shock are more consistent with those seen after monetary, consumption and investment shocks. This supports the findings of Boeri et al. (2012) as firms in the capital intensive manufacturing and construction sectors rely heaviest on financial markets to operate their businesses.

In summary, I see that financial recessions have the potential to create prolonged sluggish recoveries and cause the unemployment rate and average duration of unemployment to remain high for a significant time period after the financial shock. A closer look at particular economic series suggests that sectors most likely associated with capital financing (manufacturing and construction) are the sectors that are slowest to recover and sectors less reliant on capital financing (retail trade and service providers) show little to no distinction between financial shocks and other demand and supply shocks.

## 5 Simulations and Forecasts

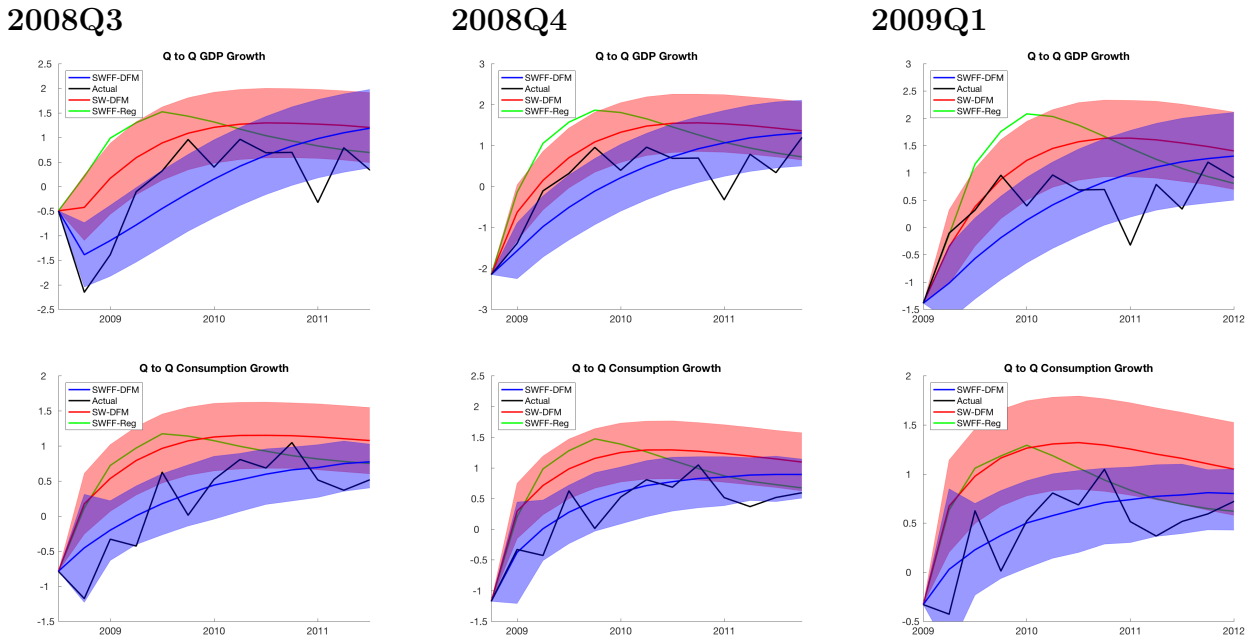
Del Negro and Schorfheide (2012) have found that the SWFF-Reg model significantly “outperformed” the SW-Reg model in regards of identifying and forecasting the output and inflation dynamics associated with the lead-up to the Great Recession and its recovery. In this section I perform a similar exercise of comparing the simulated and forecasting ability of the SW-Reg, SW-DFM and SWFF-DFM models; but instead of just focusing on output and inflation, I pay particular attention to series related to the labor and finance markets. Of course, many of these series can only be forecasted using the SWFF-DFM and SW-DFM models that were estimated in a data-rich environment.

In particular, I take the estimated posterior distributions of the models' structural parameters and loading coefficients of the  $\Lambda$  matrix and create simulated paths for the different time series for both models. I estimate the models at three different time periods, one at which all data related from 1984Q2 to 2008Q3 is available to the econometrician, one at which the econometrician can see quarterly data related to 2008Q4 and one in which they have 2009Q1 data values available to them. The models' posterior parameters are not re-estimated when the new data are revealed, instead the new values are inserted into the Kalman filter and are used as the new starting points for each of the simulations.

In total each forecast is generated by 500,000 simulations, 5,000 draws from the posterior parameter distribution and each parameter draw is simulated using 100 draws of future structural shocks for 16 quarters. In all simulations the zero lower bound is protected using shadow monetary policy shocks using an algorithm outlined by Holden and Paetz (2012).

Before looking at forecasts for macro-finance variables not inside the DSGE models, I first compare the growth forecasts of the SWFF-Reg (green), SW-DFM (red) and SWFF-DFM (blue) for real Output, Consumption and Investment against actual realized growth for these three series (black). Figures 10 and 11 show the median forecast as well as the 68% forecast posterior density intervals for the three expenditure series at three different starting times.

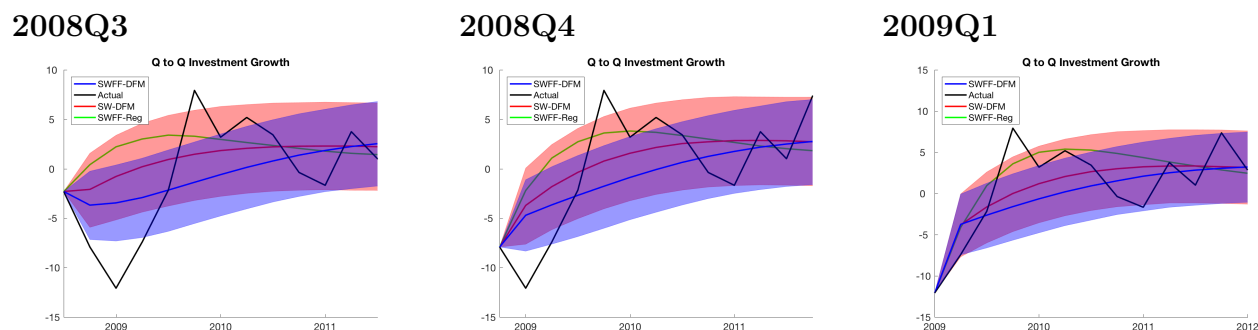
**Figure 10:** Forecasts for Quarter to Quarter Real GDP and Consumption Growth



Of note, both the DSGE-DFM models outperform the DSGE-Reg models in terms of forecast accuracy for real GDP and real Consumption growth around the Great Recession and its recovery. In addition, the SWFF-DFM model can foresee the magnitude of the Great Recession starting in 2008Q3 and foresee the sluggish growth in consumption throughout the next three years. It does overstate the decline in real GDP in the year 2009, however this may be due to the models inability to capture the unconventional fiscal and monetary policy that took place over this time period. Further, notice the overly optimistic SWFF-Reg model which predicts a quick and robust recovery.

Figure 11 shows the median forecast for real Investment at three different starting times. With regards to this variable none of the three models can foresee the depths of decline in Investment although both DSGE-DFM models predict multiple quarters of negative investment growth starting in 2008Q3. However, once the depth of the decline in Investment had been realized the SWFF-Reg model does the best job in predicting the dynamics of the real Investment recovery.

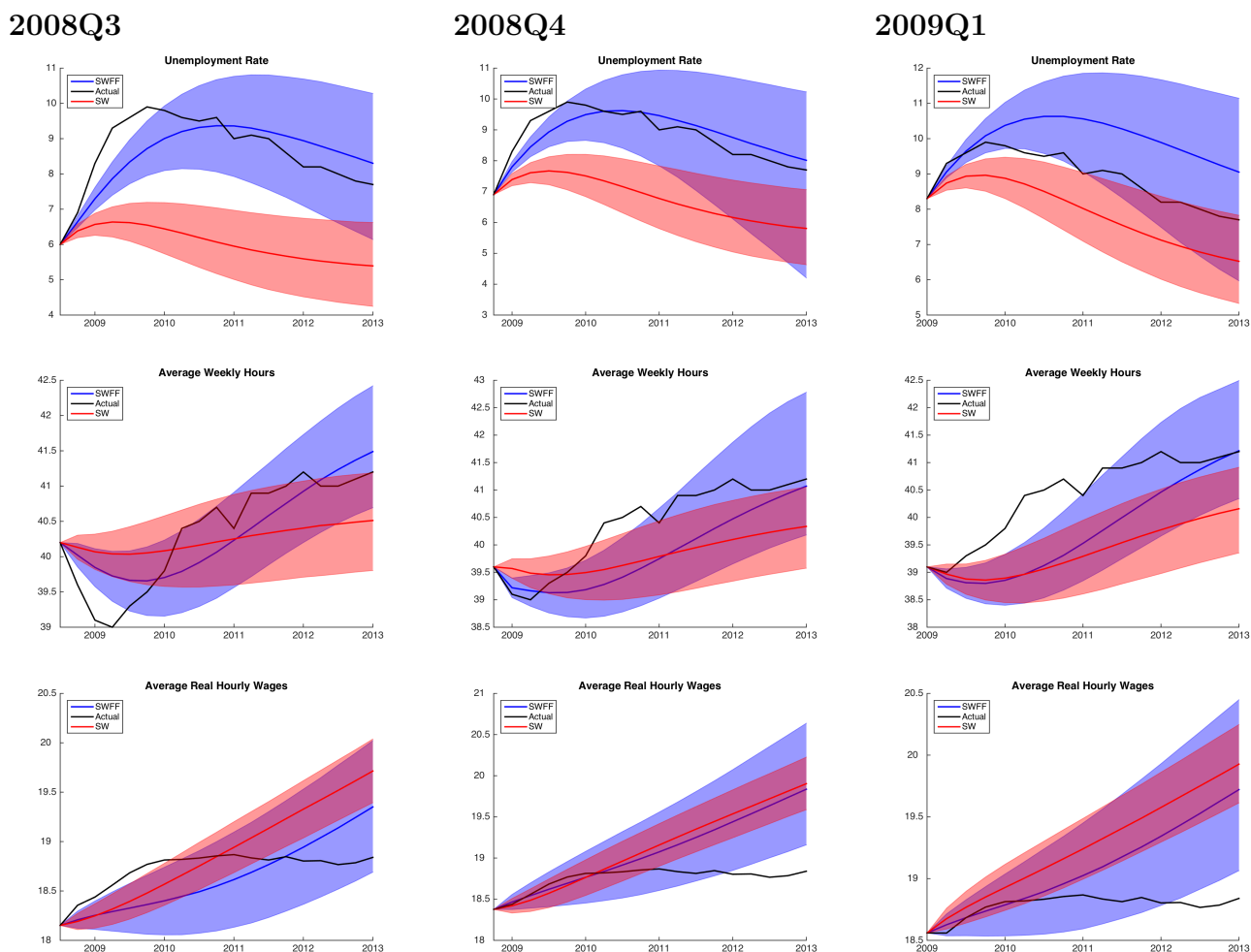
**Figure 11:** Forecasted Paths for Quarter to Quarter Real Investment Growth



Let's now look at the forecasted paths of some labor market metrics including the unemployment rate, average weekly hours and average real hourly wages. The SWFF-DFM forecasted paths are in blue and the SW-DFM forecasted paths are in red, while the actual series values are shown in black. All forecasts have been transformed into actual levels. The forecasted paths of all of these series can be found in Figure 12. Notice that the SWFF-DFM model is able to pick up the upcoming increase of the unemployment rate as early as the fall of 2008. In contrast the SW-DFM does not forecast an unemployment rate above 9% until after the 1st quarter of 2009. There is more forecast overlap between the models for

average weekly hours and average hourly wages, yet the SWFF-DFM model is still better at picking up the initial decrease in weekly hours. The stagnation of real hourly wages over the first few years after the Great Recession is not projected by either model, however, the SWFF-DFM model does predict a lower real wage when compared to the SW-DFM model.

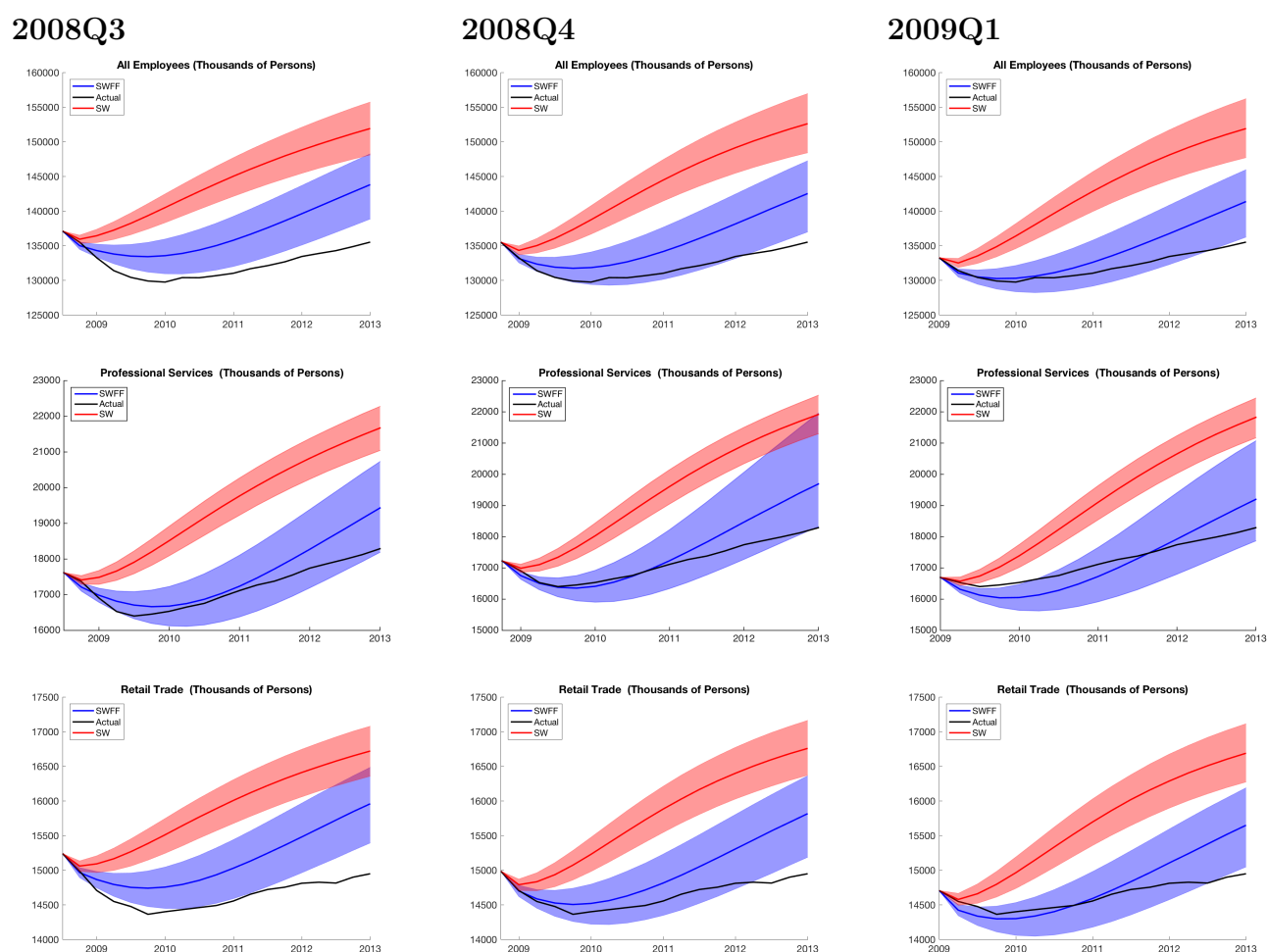
**Figure 12:** Forecasted Paths for Labor Market Metrics



When I examine the number of overall employees in the economy and the number of employees by sector in Figures 13 and 14 I find a similar story. The model with a modeled finance market (SWFF-DFM) does an impressive job of forecasting the sector employment declines. In addition, the SWFF-DFM model significantly out forecasts the SW-DFM model that does not have a modeled financial market inside its DSGE structure when it comes to overall employees and employees in the professional services, retail trade, construction,

manufacturing and wholesale trade sectors. Although the SWFF-DFM model constantly outperforms the SW-DFM model in predicting the future paths of all of these series it is still overly optimistic about the number of jobs in the economy 3-4 years into the future. This may be a result of workforce demographic changes seen around the country. Under their current construction the models have no ability to see such a demographic change as they use the population of 16 years and older (not prime-working age population) to transform variables in per capita terms.

**Figure 13:** Forecasted Paths for Labor Market Sectors



**Figure 14:** Forecasted Paths for Labor Market Sectors

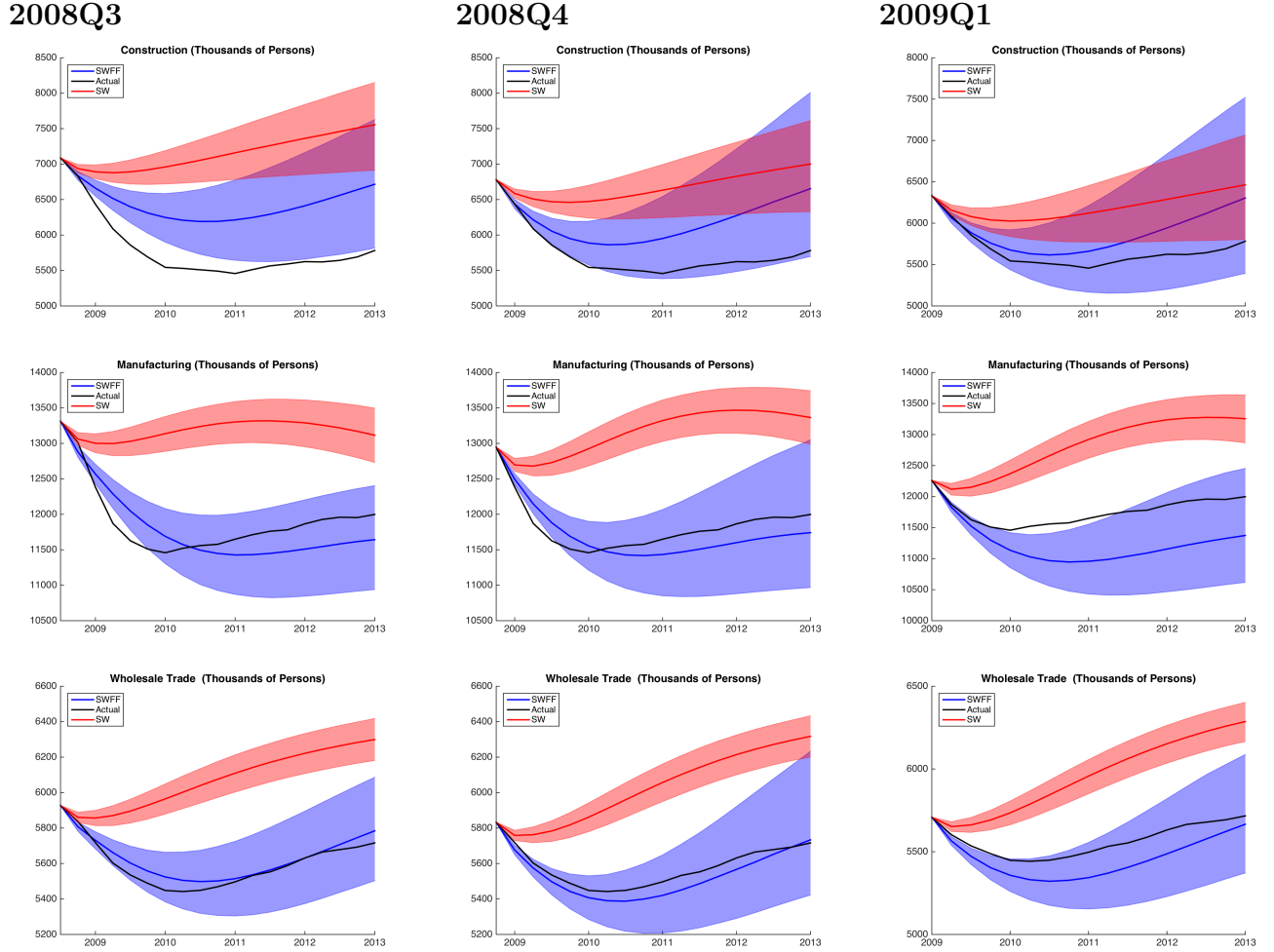
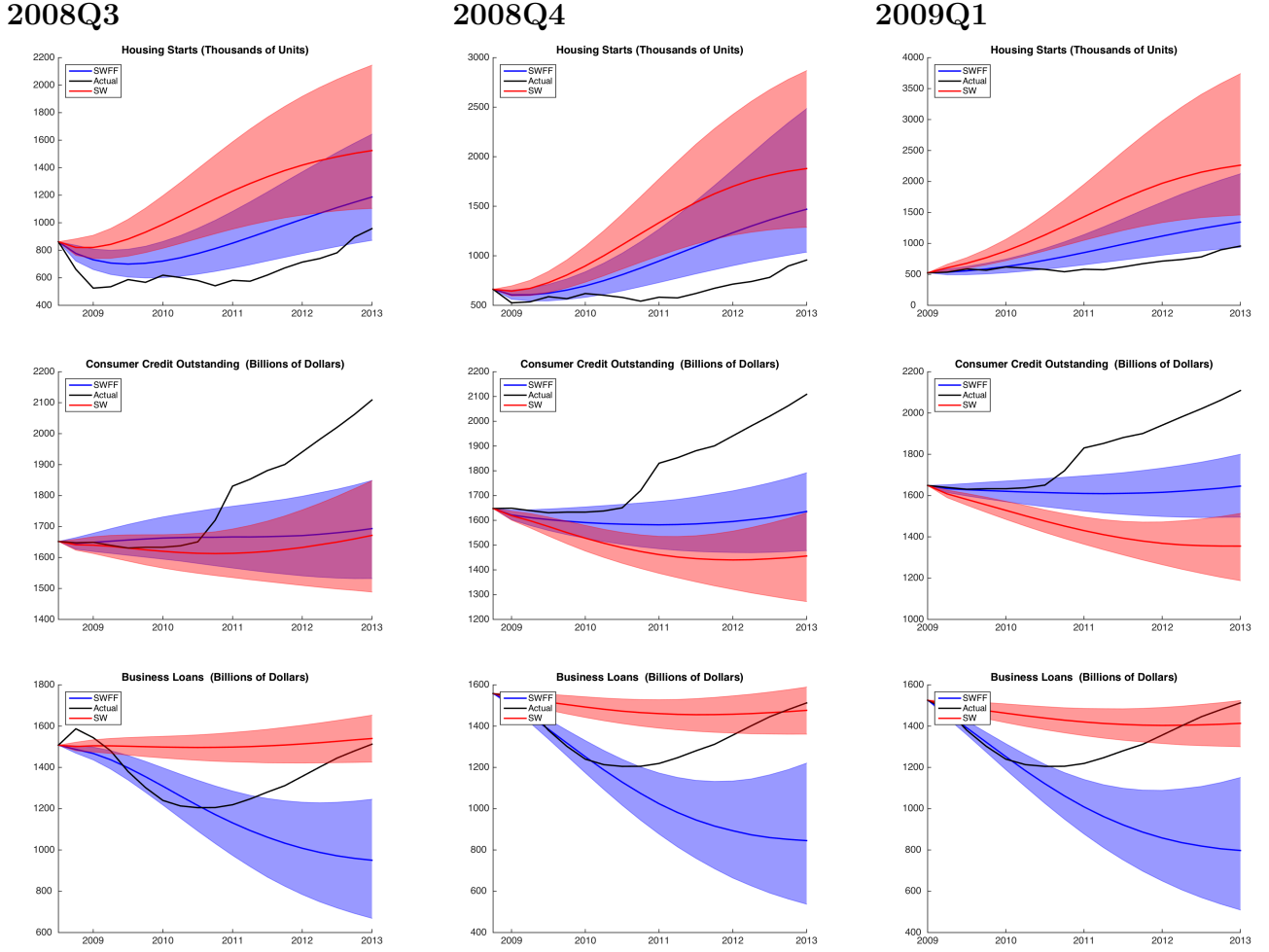


Figure 15 shows the forecasted paths of housing starts, consumer credit outstanding and business loans. Once again I see that the SWFF-DFM model soundly outperforms the SW-DFM model when it comes to housing starts. As far as consumer and business loans, the SWFF-DFM model is a good predictor of both for the first 4-6 quarters of each forecast. However, the SWFF-DFM model is unable to forecast the significant increases in both consumer and business loans that starts in the middle of 2010. One possible explanation for the increase in both could be QE2, which started in August 2010. Of course neither model has a mechanism to incorporate such a policy change.



**Figure 15:** Forecasted Paths for Financial Metrics



In summary, the SWFF-DFM model is able to see the decrease in jobs and the increase in the unemployment rate starting in 2008Q3. Additionally the SWFF-DFM model foresees the slower rate of overall jobs and jobs in particular sectors. I see that there is significant difference in the forecasted paths between the two models for the 2008-2013 time period. Yet this is not always the case for previous time periods, if I examine periods in which the financial spread was low and financial volatility was also low the forecasted paths between the models share similar posterior density intervals as can be seen in Figure 18 of the appendix. This would suggest that in addition to real output the SWFF model is better at identifying the dynamics of the labor and finance markets in times of high financial volatility.

## 5.1 Mechanisms Behind the Results

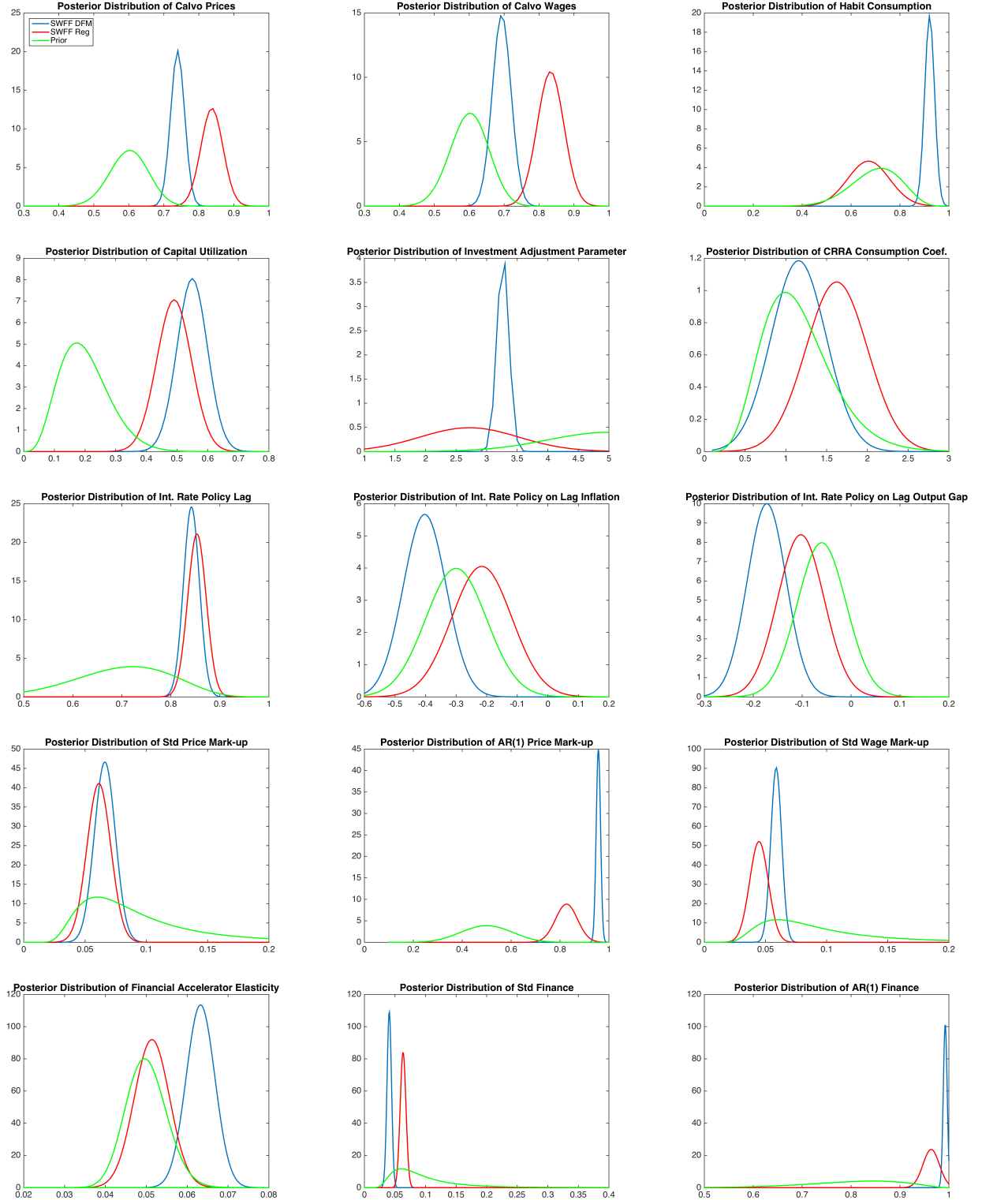
It is important to observe some characteristics and trends across the estimation techniques by examining Figure 16 and Table 4, to better understand why the SWFF-DFM model was able to foresee the output and labor dynamics associated with the Great Recession more accurately and quicker than the SWFF-Reg, SW-Reg and SW-DFM models. Figure 16 plots the posterior distributions when fitted to a normal distribution for a select number of structural parameters for the SWFF model. A few observations emerge. First, the price and wage Calvo estimates share little to no overlap between the estimation techniques. The average length of contract negotiation for prices and wages is six quarters under the DSGE-Reg estimation compared to about every three quarters in the DSGE-DFM estimation. These smaller, yet still significant, price and wage rigidities are more in line with the findings of Klenow and Kryvtsov (2008) who examined monthly price changes by industry and found that the mean price duration is about 7 months. The parameter that governs habit formation consumption substantially increases in the DSGE-DFM estimation for both the SW and SWFF models when compared to its estimate under DSGE-Reg estimation. This helps explain why the SWFF-DFM model is able to forecast the sluggish growth in consumption during the recovery shown in Figure 10.

Taylor Rule policy parameters are found to be more responsive to lagged inflation and the lagged output gap when estimated in the data-rich environment implying more inertia and persistence in the model. The policy parameters regarding the contemporaneous output gap and inflation levels are estimated to be less responsive in the data-rich environment.

Many of the parameters linked to the exogenous shocks of the model remain similar across the estimation techniques of the SWFF model. However, price and wage mark-up shocks are estimated to be much more persistent in the SWFF-DFM estimation technique. The presence of many other price and wage indexes, including oil prices, drive this result as different inflation dynamics are needed to encapsulate many of them.

The parameters that preside over the financial accelerator also change when estimated in a data-rich environment. There is more inertia in the financial accelerator as the spread elasticity is found to be larger and the finance shock is found to be smaller but much more

**Figure 16:** Posterior Distribution Estimates of Structural Parameters in SWFF



persistence. The extra estimated persistence in nearly all structural shocks in the SWFF-DFM coupled with its modeled financial market can explain why the SWFF-DFM was able anticipate the slow recovery in GDP, consumption and sector employment after large negative financial, investment and productivity shocks seen in 2008.

Figure 17 compares the forecasts of All Employees around the Great Recession under different estimation concepts. The top row of plots, forecasts of All Employees using the DSGE-DFM approach discussed throughout the paper. The second row of plots, forecasts All Employees using the DSGE-Reg method and regressing historical employment data with the estimated states of the DSGE-Reg models.<sup>11</sup> I refer to models estimated in this fashion as DSGE-OLS. I find that simply estimating the structural parameters of the DSGE model using only core macroeconomic variables and then regressing the estimated states on employment data does not provide an accurate forecast of All Employees. This highlights that utilizing the large set of macro-finance data series in the estimation of the structural parameters inside the DSGE model can generate more accurate forecasts of core and non-core macroeconomic data series.

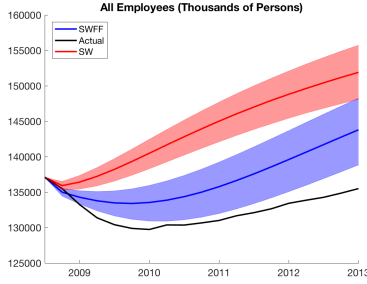
The third and fourth row of plots in Figure 17 underscore the importance of certain types of data. Both are DSGE-DFM models estimated using the approach discussed throughout the paper, however, row three does not include the series grouped in the Output Components category in the  $X_t$  data matrix and row four does not include the series grouped in the Price and Wages Indexes category in the  $X_t$  data matrix. The absence of either of these series significantly undermines the improvement seen in forecasting All Employees for the SWFF-DFM model when it is estimated using all the data series shown in the Appendix A. Without multiple Output and Price indicators the SWFF-DFM model is unable to generate the additional shock persistence needed to resemble the dynamics of the Great Recession as is the case when these output and price indicators are included in the  $X_t$  data matrix. I find similar patterns when I look at the forecasts of housing starts, business loans, consumer loans and employment by sector.

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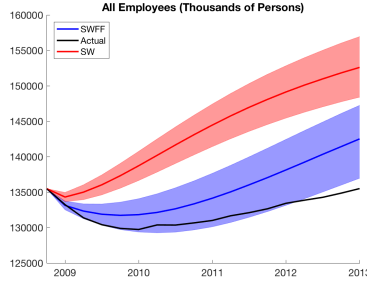
<sup>11</sup>Recall this is how the DSGE-DFM algorithm outlined in Section 3.2 is initialized.

**Figure 17:** Forecasts for All Employees Under Different Estimation Concepts

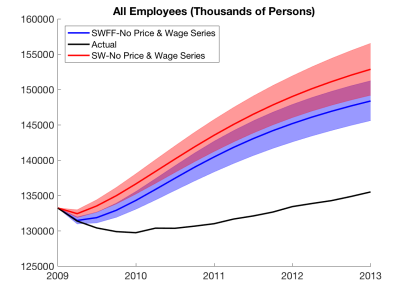
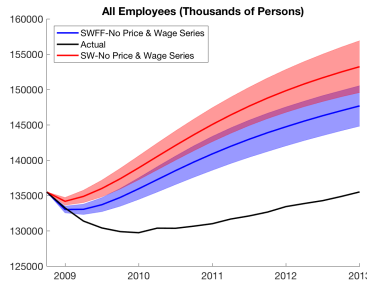
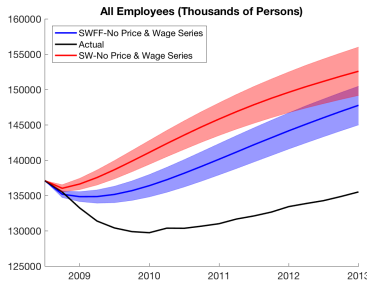
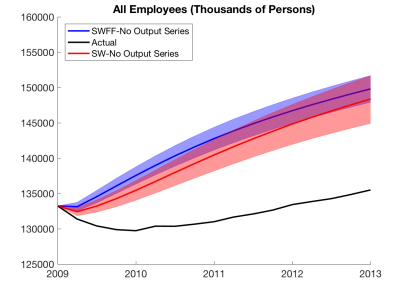
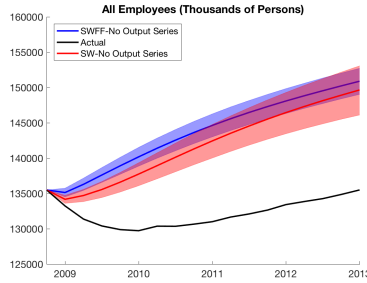
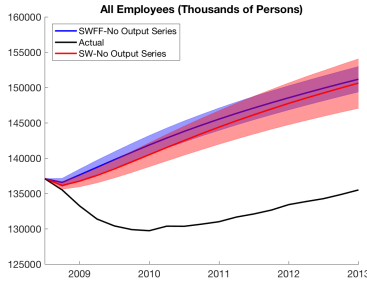
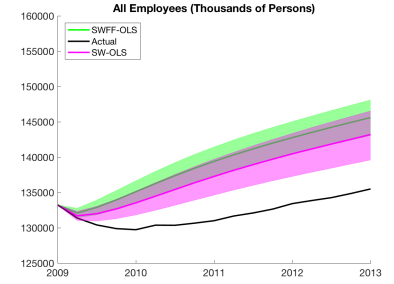
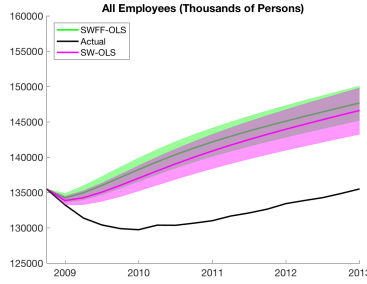
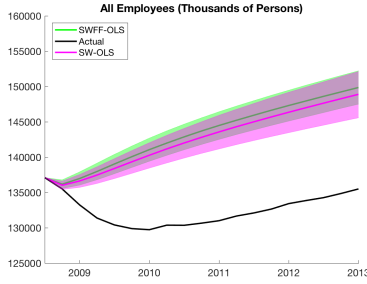
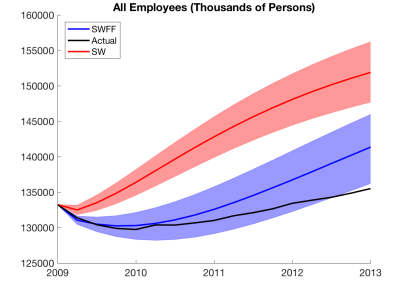
**2008Q3**



**2008Q4**



**2009Q1**



## 6 Conclusion

In this paper, the Smets and Wouters (2003, 2007) New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model augmented with a financial accelerator (SWFF) is estimated using a large set of economic and financial series following the work of Boivin and Giannoni (2006) and Kryshko (2011). To explore the economic and labor market effects of

various exogenous shocks, I examine structural impulse response functions (IRF's) for series that are usually not obtainable inside DSGE models or only obtainable if embedded in a dynamic factor model with little or no theoretical interpretation of the original shock that they are generated by. However, the Boivin and Giannoni estimation technique (DSGE-DFM) creates a structural foundation of what type of initial shock has created the disturbance. An examination at calibrated IRF's suggests that financial shocks have very different effects on the labor, finance and investment markets when compared to their structural counter-parts of monetary, consumer, government and supply shocks. Most notably, manufacturing and construction sectors are the very sectors that are slowest to recover from a financial shock. Further, the decreases in real investment, residential investment, exports and new orders are larger and last longer after negative financial shocks.

I also find that identical decreases of GDP generated by different structural shocks of the SWFF model creates different magnitudes in the change of the overall unemployment rate. These results suggest that the relationship between unemployment and GDP growth implied by Okun's Law may be state-dependent.

Comparing the original Smets and Wouters (2003, 2007) model (SW) and SWFF DSGE-DFM models, I find that the SWFF-DFM model is better in capturing the dynamics of many economic series including output, consumption and many labor market metrics around the time of the Great Recession and its ensuing recovery. This result suggests that a structural DSGE model embedded with a modeled financial market and estimated in a data-rich environment would have predicted the output and labor market severity of the Great recession and its aftermath as early as the summer of 2008. Finally, I believe the continuing advancements in computational programming and the ever growing number of macroeconomic and financial series available allows DSGE-DFM estimation to be a bountiful area of future research.

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## A Appendix: Data and Transformations

| Kryshko Shorthand            | FRED Code        | Trans*          | Long Description   | Used in Reg Estimation |
|------------------------------|------------------|-----------------|--|------------------------|
| <b>Core Sets</b>             |                  |                 |  |                        |
| <b>Core Output</b>           |                  |                 |  |                        |
| 1                            | RGDP             | GDPC1           | 2 Real GDP   | *                      |
| 2                            | IP_TOTAL         | INDPRO          | 2 Industrial Production Index:total                        |                        |
| 3                            | RGDI             | A261RX1Q020SBEA | 2 Real Domestic Income                                     |                        |
| <b>Core Inflation</b>        |                  |                 |  |                        |
| 4                            | PGDP             | GDPDEF          | 3 GDP Price deflator                                       | *                      |
| 5                            | PCED             | PCECTPI         | 3 PCE_ALL Price deflator                                   |                        |
| 6                            | CPI_ALL          | CPIAUCSL        | 3 CPI_ALL Price index                                      |                        |
| <b>Core Consumption</b>      |                  |                 |  |                        |
| 7                            | RCONS            | PCECC96         | 2 Real Personal Consumption Expenditures                   | *                      |
| <b>Core Investment</b>       |                  |                 |  |                        |
| 8                            | RINV             | GDPI            | 2 Real Private Domestic Investment                         | *                      |
| <b>Core Wages</b>            |                  |                 |  |                        |
| 9                            | RWAGE            | AHETPI          | 4 Real Average Hourly wages:production:total private       | *                      |
| <b>Core Labor Employment</b> |                  |                 |  |                        |
| 10                           | HOURS            | HOANBS          | 2 Hours Worked   | *                      |
| 11                           | EMP_CES          | PAYEMS+USGOVT   | 2 Employees:Total Nonfarm                                  |                        |
| 12                           | EMP_CPS          | CE160V          | 2 Civilian Labor Force:Employed, Total                     |                        |
| <b>Core Interest Rate</b>    |                  |                 |  |                        |
| 13                           | FedFunds         | FEDFUNDS        | 0 Federal Funds Rate (effective)                           | *                      |
| 14                           | Tbill_3m         | TB3MS           | 0 Interest Rate U.S. Treasury bill 3 month                 |                        |
| 15                           | AAABond          | AAA             | 0 Bond Yield: Moody's AAA corporate                        |                        |
| <b>Core Spread*</b>          |                  |                 |  |                        |
| 16                           | SFYBAAC          | BAA-GS10        | 0 Spread of BAA corporate yield to 10 year Treasury        | *                      |
| 17                           | SFYAAAC          | AAA-GS10        | 0 Spread of AAA corporate yield to 10 year Treasury        |                        |
| <b>Non-Core Sets</b>         |                  |                 |  |                        |
| <b>Output Components</b>     |                  |                 |  |                        |
| 18                           | IP_FINAL         | IPS299          | 2 Industrial Production Index:final products               |                        |
| 19                           | IP_CONS_DBLE     | IPDCONGD        | 2 Industrial Production Index:Durable Consumer Goods       |                        |
| 20                           | IP_CONS_NONDBLE  | IPNCONGD        | 2 Industrial Production Index:NonDurable Consumer Goods    |                        |
| 21                           | IP_BUS_EQPT      | IPBUSEQ         | 2 Industrial Production Index:Business Equipment           |                        |
| 22                           | IP_DRBLE_MATS    | IPDMAT          | 2 Industrial Production Index:Durable Goods Materials      |                        |
| 23                           | IP_NONDRBLE_MATS | IPNMAT          | 2 Industrial Production Index:NonDurable Goods Materials   |                        |
| 24                           | IP_MFG           | IPMAN           | 2 Industrial Production Index:Manufacturing                |                        |
| 25                           | IP_FUELS         | IPUTIL          | 2 Industrial Production Index:Fuels                        |                        |
| 26                           | PMP              | NAPMPI          | 0 NAPM Production index                                    |                        |
| 27                           | RCONS_DRBLE      | DDURRA3Q086SBEA | 2 Real Personal Consumption Expenditures index:Durables    |                        |
| 28                           | RCONS_NONDRBLE   | DNDGRA3Q086SBEA | 2 Real Personal Consumption Expenditures index:NonDurables |                        |
| 29                           | RCONS_SERV       | DSERRA3Q086SBEA | 2 Real Personal Consumption Expenditures index:Sevices     |                        |
| 30                           | REXPORTS         | B020RA3Q086SBEA | 2 Real Exports Quantity Index                              |                        |
| 31                           | RIMPORTS         | B255RA3Q086SBEA | 2 Real Imports Quantity Index                              |                        |
| 32                           | RGOV             | B823RA3Q086SBEA | 2 Real Government Consumption & Investment Quantity Index  |                        |
| <b>Labor Market</b>          |                  |                 |  |                        |
| 33                           | EMP_Mining       | USMINE          | 2 Employees:Mining & Logging                               |                        |
| 34                           | EMP_CONST        | USCONS          | 2 Employees:Construction                                   |                        |
| 35                           | EMP_MFG          | MANEMP          | 2 Employees:Manufacturing                                  |                        |
| 36                           | EMP_SERVICES     | SRVPRD          | 2 Employees:Service Providing                              |                        |
| 37                           | EMP_TTU          | USTPU           | 2 Employees:Trade, Transportation, Utilities               |                        |
| 38                           | EMP_WHOLESALE    | USWTRADE        | 2 Employees:Wholesale Trade                                |                        |
| 39                           | EMP_RETAIL       | USTRADE         | 2 Employees:Retail Trade                                   |                        |
| 40                           | EMP_FIN          | USFIRE          | 2 Employees:Financial Activities                           |                        |
| 41                           | EMP_GOVT         | USGOVT          | 2 Employees:Government                                     |                        |
| 42                           | EMP_PROSERV      | USPBS           | 2 Employees:Professional Services                          |                        |
| 43                           | EMP_LEISURE      | USLAH           | 2 Employees:Leisure & Hospitality                          |                        |
| 44                           | URATE            | UNRATE          | 0 Unemployment Rate  |                        |
| 45                           | U_DURATION       | UEMPMEAN        | 0 Average Duration of Unemployment (weeks)                 |                        |
| 46                           | U_5WKS           | UEMPLT5         | 2 Unemployment Duration:Persons:Less than 5 Weeks          |                        |
| 47                           | U_5_14WKS        | UEMP5TO14       | 2 Unemployment Duration:Persons:5-14 Weeks                 |                        |
| 48                           | U_15_26WKS       | UEMP15T26       | 2 Unemployment Duration:Persons:15-26                      |                        |

|                                 |                  |                  |   |  |
|---------------------------------|------------------|------------------|---|--|
| 49                              | U_M27WKS         | UEMP27OV         | 2 | Unemployment Duration:Persons:27 weeks +                       |
| 50                              | HOURS_AVG        | CES0600000007    | 0 | Average Weekly Hours:Goods Producing                           |
| 51                              | HOURS_AVG_OT     | AWOTMAN          | 0 | Average Weekly Overtime Hours:Manufacturing                    |
| <b>Housing Market</b>           |                  |                  |   |  |
| 52                              | HSTARTS_NE       | HOUSTNE          | 1 | Housing Starts:Northeast                                       |
| 53                              | HSTARTS_MW       | HOUSTMW          | 1 | Housing Starts:Midwest   |
| 54                              | HSTARTS_SOU      | HOUSTS           | 1 | Housing Starts:South   |
| 55                              | HSTARTS_WST      | HOUSTW           | 1 | Housing Starts:West  |
| 56                              | RRRESINV         | B011RA3Q086SBEA  | 2 | Real Private Domestic Investment:Residential Quantity Index    |
| <b>Financial Market</b>         |                  |                  |   |  |
| 57                              | SFYGM6           | TB6MS-TB3MS      | 0 | Spread of 6 month Tbill to 3 month Tbill                       |
| 58                              | SFYGT1           | GS1-TB3MS        | 0 | Spread of 1 year Treasury to 3 month Tbill                     |
| 59                              | SFYGT10          | GS10-TB3MS       | 0 | Spread of 10 year Treasury to 3 month Tbill                    |
| 60                              | TOT_RES          | TOTRESNS         | 2 | Total Reserves of Depository Institutions                      |
| 61                              | TOT_RES_NB       | NONBORRES        | 5 | Total Reserves Of Depository Institutions, Nonborrowed         |
| 62                              | BUS_LOANS        | BUSLOANS         | 2 | Commercial and Industrial Loans at All Commercial Banks        |
| 63                              | CONS_CREDIT      | NONREVSL         | 2 | Total Nonrevolving Credit Owned and Securitized, Outstanding   |
| 64                              | SP500            | SP500            | 3 | S&P 500 Stock Price Index                                      |
| 65                              | DJIA             | DJIA             | 3 | Dow Jones Industrial Average                                   |
| <b>Exchange Rates</b>           |                  |                  |   |  |
| 66                              | EXR_US           | TWEXMMTH         | 3 | Trade Weighted U.S. Dollar Index: Major Currencies             |
| 67                              | EXR_SW           | EXSZUS           | 3 | Switzerland / U.S. Foreign Exchange Rate                       |
| 68                              | EXR_JAN          | EXJPUS           | 3 | Japan / U.S. Foreign Exchange Rate                             |
| 69                              | EXR_UK           | EXUSUK           | 3 | U.S. / U.K. Foreign Exchange Rate                              |
| 70                              | EXR_CAN          | EXCAUS           | 3 | Canada / U.S. Foreign Exchange Rate                            |
| <b>Investment</b>               |                  |                  |   |  |
| 71                              | NAPMI            | NAPM             | 0 | Purchasing Managers Index                                      |
| 72                              | NAPM_NEW_ORDERS  | NAPMNOI          | 0 | NAPM New Orders Index  |
| 73                              | NAPM_SUP_DEL     | MAPMSDI          | 0 | NAPM Supplier Deliveries                                       |
| 74                              | NAPM_INVENTORIES | NAPMII           | 0 | NAPM Inventories Index   |
| 75                              | RNONRESINV       | B009RA3Q086SBEA  | 2 | Real private fixed investment: Nonresidential quantity index   |
| <b>Price &amp; Wage Indexes</b> |                  |                  |   |  |
| 76                              | RAHE_CONST       | CES3000000008    | 4 | Real Avg. Hourly wages:construction (Deflated w/GDP Deflator)  |
| 77                              | RAHE_MFG         | CES3000000008    | 4 | Real Avg. Hourly wages:manufacturing (Deflated w/GDP Deflator) |
| 78                              | RCOMP_HR         | COMPRNFB         | 4 | Real Compensation Per Hour (index)                             |
| 79                              | ULC              | ULCNFB           | 4 | Unit Labor Cost (index)  |
| 80                              | CPI_CORE         | CPILFESL         | 3 | CPI:Less food and energy                                       |
| 81                              | PCED_DUR         | DDURRA3Q086SBEA  | 3 | PCE:Durable goods price index                                  |
| 82                              | PCED_NDUR        | DNDGRA3Q086SBEA  | 3 | PCE:NonDurable goods price index                               |
| 83                              | PCED_SERV        | DSERRG3Q086SBEA  | 3 | PCE:Services price index                                       |
| 84                              | PINV_GDP         | GPDICTPI         | 3 | Gross private domestic investment price index                  |
| 85                              | PINV_NRES_STRUCT | B009RG3Q086SBEA  | 3 | GPDI:price index:structures                                    |
| 86                              | PINV_NRES_EQP    | B010RG3Q086SBEA  | 3 | GPDI:price index:Equipment and software                        |
| 87                              | PINV_RES         | B011RG3Q086SBEA  | 3 | GPDI:price index:Residential                                   |
| 88                              | PEXPORTS         | (B020RG3Q086SBEA | 3 | GDP:Exports Price Index  |
| 89                              | PIMPORTS         | B021RG3Q086SBEA  | 3 | GDP:Imports Price Index  |
| 90                              | PGOV             | B822RG3Q086SBEA  | 3 | Government Consumption and gross investment price index        |
| 91                              | P_COM            | PPIACO           | 3 | PPI:All commodities price index                                |
| 92                              | P_OIL            | PPICEM/PCEPILFE  | 3 | PPI:Crude (Divided by PCE Core)                                |
| <b>Other</b>                    |                  |                  |   |  |
| 93                              | UTL11            | MCUMFN           | 0 | Capacity Utilization-Manufacturing                             |
| 94                              | LABOR_PROD       | OPHNFB           | 4 | Output per hour all persons:business sector index              |
| 95                              | UMICH_CONS       | UMCSENT          | 1 | University of Michigan Consumer Expectations                   |
| 96                              | M_1              | M1SL             | 2 | M1 Money stock   |
| 97                              | M_2              | M2SL             | 2 | M2 Money stock   |

\*Transformation codes are described in the data transformation rubric

Note: Since there is no Spread variable in the SW Model, data set 16 is not used in the SW-Reg estimation and data sets 16 and 17 are moved to the Financial Market grouping for SW-DFM estimation

## Data Transformation Rubric

| Code | Description                       |
|------|-----------------------------------|
| 0    | Demeaned                          |
| 1    | Log() and demeaned                |
| 2    | Linear detrended Log() per capita |
| 3    | Log() differenced and demeaned    |
| 4    | Detrended Log()                   |
| 5    | Detrended per capita level        |

Note: All per capita variables are calculated using the adult population series. (CNP16OV)

### Measurement Equations for Reg Estimation

The measurement equation (3.2) is specified as follows where the 8th row is omitted for the SW model:

$$\begin{bmatrix} \text{RGDP} \\ \text{PGDP} \\ \text{RCONS} \\ \text{RINV} \\ \text{RWAGE} \\ \text{HOURS} \\ \text{FedFunds} \\ \text{SFYBAAC}/4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \dots & 0 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ c_t \\ I_t \\ w_t \\ L_t \\ R_t \\ S_t \\ \vdots \end{bmatrix}$$

## B Appendix: Tables & Figures

Table 4: Posterior Estimates of SWFF Model

|  | Regular Estimation |        |        | DSGE-DFM Estimation |        |        |
|--|--------------------|--------|--------|---------------------|--------|--------|
|  | Mean               | 5%     | 95%    | Mean                | 5%     | 95%    |
| <b>Structural Parameters</b>                             |                    |        |        |                     |        |        |
| $\psi$   | 0.491              | 0.414  | 0.595  | 0.550               | 0.471  | 0.649  |
| $\iota_p$  | 0.261              | 0.099  | 0.495  | 0.106               | 0.040  | 0.181  |
| $\iota_w$  | 0.250              | 0.128  | 0.389  | 0.426               | 0.240  | 0.676  |
| $\xi_p$  | 0.837              | 0.783  | 0.887  | 0.739               | 0.708  | 0.776  |
| $\xi_w$  | 0.833              | 0.759  | 0.882  | 0.693               | 0.654  | 0.740  |
| $\nu_l$  | 1.782              | 1.127  | 2.545  | 1.244               | 0.785  | 1.849  |
| $\sigma_c$   | 1.624              | 1.057  | 2.323  | 1.157               | 0.725  | 1.843  |
| $h$  | 0.672              | 0.525  | 0.806  | 0.921               | 0.888  | 0.951  |
| $\phi$   | 0.467              | 0.219  | 0.760  | 0.176               | 0.052  | 0.380  |
| $S$  | 2.716              | 1.471  | 4.138  | 3.267               | 3.074  | 3.394  |
| $\chi$   | 0.051              | 0.044  | 0.059  | 0.063               | 0.057  | 0.069  |
| <b>Policy Parameters</b>                                 |                    |        |        |                     |        |        |
| $r_{\pi_1}$  | 2.196              | 1.832  | 2.602  | 1.539               | 1.397  | 1.706  |
| $r_{y_1}$  | 0.336              | 0.235  | 0.443  | 0.131               | 0.070  | 0.209  |
| $r_{\pi_2}$  | -0.216             | -0.383 | -0.056 | -0.403              | -0.536 | -0.289 |
| $r_{y_2}$  | -0.103             | -0.179 | -0.024 | -0.172              | -0.252 | -0.110 |
| $\rho$   | 0.853              | 0.821  | 0.883  | 0.842               | 0.810  | 0.864  |
| <b>Exogenous Processes AR(1) Parameters</b>              |                    |        |        |                     |        |        |
| $\rho_a$   | 0.910              | 0.877  | 0.940  | 0.944               | 0.928  | 0.955  |
| $\rho_b$   | 0.755              | 0.623  | 0.863  | 0.726               | 0.673  | 0.776  |
| $\rho_G$   | 0.971              | 0.951  | 0.987  | 0.867               | 0.838  | 0.890  |
| $\rho_I$   | 0.664              | 0.549  | 0.766  | 0.843               | 0.765  | 0.913  |
| $\rho_F$   | 0.964              | 0.932  | 0.986  | 0.993               | 0.985  | 0.998  |
| $\rho_p$   | 0.826              | 0.745  | 0.891  | 0.957               | 0.941  | 0.969  |
| $\rho_w$   | 0.600              | 0.432  | 0.781  | 0.911               | 0.853  | 0.952  |
| <b>Exogenous Processes Standard Deviation Parameters</b> |                    |        |        |                     |        |        |
| $\sigma_a$   | 0.487              | 0.431  | 0.550  | 0.428               | 0.343  | 0.500  |
| $\sigma_b$   | 0.094              | 0.063  | 0.131  | 0.026               | 0.019  | 0.034  |
| $\sigma_G$   | 0.327              | 0.290  | 0.372  | 0.230               | 0.179  | 0.289  |
| $\sigma_r$   | 0.127              | 0.111  | 0.145  | 0.130               | 0.119  | 0.148  |
| $\sigma_I$   | 0.955              | 0.801  | 1.129  | 0.241               | 0.192  | 0.308  |
| $\sigma_F$   | 0.063              | 0.056  | 0.072  | 0.041               | 0.035  | 0.047  |
| $\sigma_p$   | 0.061              | 0.047  | 0.078  | 0.066               | 0.052  | 0.081  |
| $\sigma_w$   | 0.045              | 0.033  | 0.058  | 0.059               | 0.051  | 0.065  |

**Table 5:** Posterior Estimates of SW Model

|  | <b>Regular Estimation</b> |        |        | <b>DSGE-DFM Estimation</b> |        |        |
|--|---------------------------|--------|--------|----------------------------|--------|--------|
|  | Mean                      | 5%     | 95%    | Mean                       | 5%     | 95%    |
| <b>Structural Parameters</b>                             |                           |        |        |                            |        |        |
| $\psi$   | 0.345                     | 0.208  | 0.497  | 0.284                      | 0.155  | 0.442  |
| $\iota_p$  | 0.261                     | 0.102  | 0.493  | 0.229                      | 0.093  | 0.411  |
| $\iota_w$  | 0.223                     | 0.108  | 0.356  | 0.442                      | 0.210  | 0.672  |
| $\xi_p$  | 0.838                     | 0.787  | 0.885  | 0.689                      | 0.609  | 0.766  |
| $\xi_w$  | 0.853                     | 0.804  | 0.888  | 0.756                      | 0.634  | 0.828  |
| $\nu_l$  | 2.009                     | 1.307  | 2.880  | 1.363                      | 0.729  | 2.225  |
| $\sigma_c$   | 1.678                     | 1.115  | 2.316  | 1.233                      | 0.710  | 1.922  |
| $h$  | 0.688                     | 0.552  | 0.816  | 0.910                      | 0.852  | 0.954  |
| $\phi$   | 0.445                     | 0.201  | 0.750  | 0.128                      | 0.036  | 0.254  |
| $S$  | 5.348                     | 3.841  | 6.898  | 5.243                      | 4.560  | 6.104  |
| <b>Policy Parameters</b>                                 |                           |        |        |                            |        |        |
| $r_{\pi_1}$  | 2.161                     | 1.775  | 2.556  | 2.107                      | 1.744  | 2.498  |
| $r_{y_1}$  | 0.345                     | 0.238  | 0.460  | 0.206                      | 0.116  | 0.291  |
| $r_{\pi_2}$  | -0.222                    | -0.383 | -0.063 | -0.231                     | -0.383 | -0.085 |
| $r_{y_2}$  | -0.084                    | -0.166 | -0.005 | -0.166                     | -0.238 | -0.093 |
| $\rho$   | 0.867                     | 0.835  | 0.896  | 0.831                      | 0.796  | 0.860  |
| <b>Exogenous Processes AR(1) Parameters</b>              |                           |        |        |                            |        |        |
| $\rho_a$   | 0.911                     | 0.879  | 0.939  | 0.945                      | 0.901  | 0.979  |
| $\rho_b$   | 0.772                     | 0.654  | 0.864  | 0.755                      | 0.671  | 0.821  |
| $\rho_G$   | 0.974                     | 0.956  | 0.987  | 0.968                      | 0.949  | 0.989  |
| $\rho_I$   | 0.710                     | 0.593  | 0.813  | 0.848                      | 0.785  | 0.906  |
| $\rho_p$   | 0.827                     | 0.748  | 0.890  | 0.600                      | 0.418  | 0.734  |
| $\rho_w$   | 0.524                     | 0.381  | 0.684  | 0.588                      | 0.415  | 0.886  |
| <b>Exogenous Processes Standard Deviation Parameters</b> |                           |        |        |                            |        |        |
| $\sigma_a$   | 0.500                     | 0.442  | 0.567  | 0.209                      | 0.155  | 0.277  |
| $\sigma_b$   | 0.085                     | 0.056  | 0.120  | 0.036                      | 0.023  | 0.053  |
| $\sigma_G$   | 0.322                     | 0.287  | 0.362  | 0.292                      | 0.217  | 0.353  |
| $\sigma_r$   | 0.125                     | 0.110  | 0.142  | 0.119                      | 0.104  | 0.139  |
| $\sigma_I$   | 0.737                     | 0.603  | 0.881  | 0.263                      | 0.214  | 0.317  |
| $\sigma_q$   | 0.104                     | 0.039  | 0.244  | 0.583                      | 0.467  | 0.713  |
| $\sigma_p$   | 0.061                     | 0.047  | 0.078  | 0.098                      | 0.075  | 0.125  |
| $\sigma_w$   | 0.048                     | 0.036  | 0.060  | 0.106                      | 0.070  | 0.150  |



Figure 18: Forecasted Paths of the Mid-1990's

